Contents

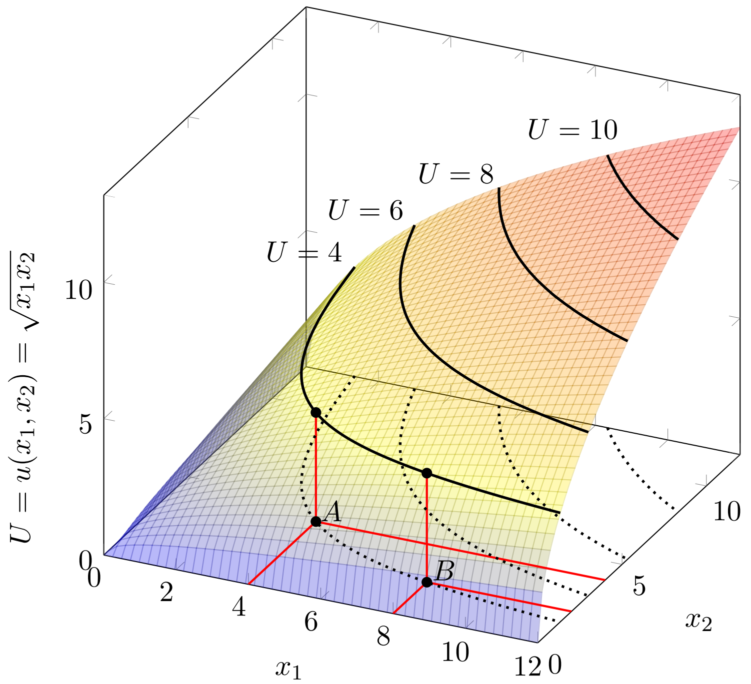
Ch. 4. Utility

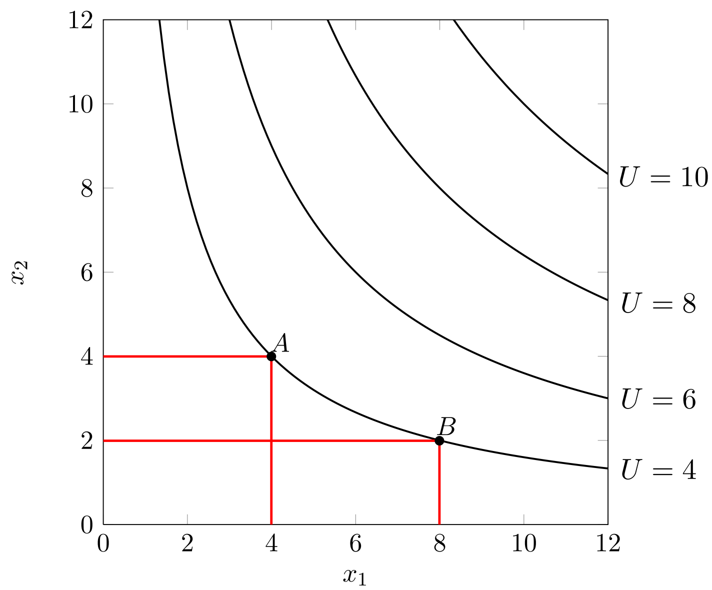
I. Utility function: An assignment of real number $u(x) \in \mathbb{R}$ to each bundle$x$

A. We say that u *represents* $\succ$ if the following holds:

$$x \succ y \text{ if and only if } u(x) > u(y)$$

– An indifference curve is the set of bundles that give the same level of utility:





B. Ordinal utility

III. Application: labor supply

– $\begin{cases} C: \text{ Consumption good} \\ p: \text{ Price of consumption good} \\ \ell: \text{ Leisure time; } \overline{L}: \text{ endowment of time} \\ w: \text{ Wage} = \text{price of leisure} \\ M: \text{ Non-labor income} \\ \overline{C} \equiv M/P: \text{ Consumption available when being idle} \end{cases}$

– $U(C, \ell)$: Utility function, increasing in both C and ℓ

– $L = \overline{L} - \ell$, labor supply

A. Budget constraint and optimal labor supply

$$pC = M + wL \Leftrightarrow M = pC - wL = pC - w(\overline{L} - \ell) \Leftrightarrow pC + wl = M + w\overline{L} = \underbrace{p\overline{C} + w\overline{L}}_{\text{value of endowment}}$$

e.g.) Assume $U(C, l) = C^a \ell^{1-a}$, $0 < a < 1$, $M = 0$, and $\overline{L} = 16$, and derive the labor supply curve

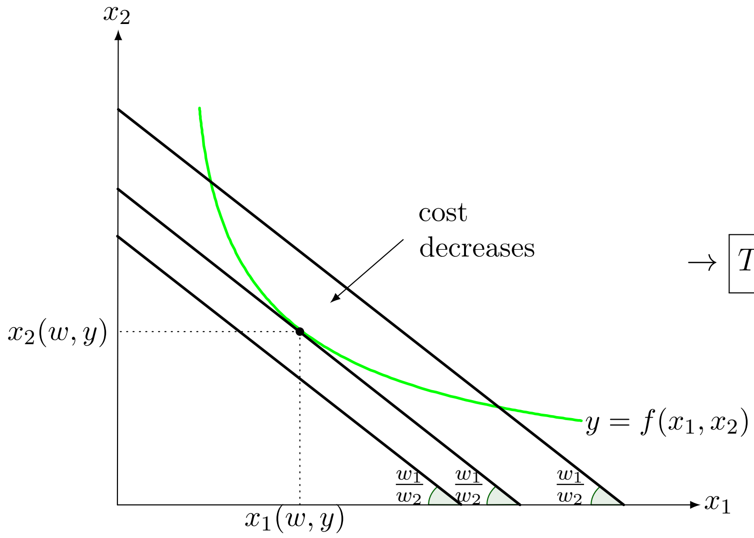
B. Changes in wage: $w < w'$

Ch. 20. Cost Minimization

I. Cost minimization: Minimize the cost of producing a given level of output y

$$\min_{x_1,x_2} w_1x_1 + w_2x_2 \text{ subject to } (x_1, x_2) \in Q(y) \text{ (i.e. } f(x_1, x_2) = y)$$

A. Tangent solution: Consider *iso-cost line* for each cost level C, $w_1x_1+w_2x_2 = C$; and find the *lowest* iso-cost line that meets the isoquant curve



$\rightarrow$ $\boxed{TRS = \frac{MP_1}{MP_2} = \frac{w_1}{w_2}}$

$$\rightarrow \begin{cases} x_1(w, y), x_2(w, y): \text{ Conditional factor demand function} \\ c(w, y) = w_1x_1(w, y) + w_2x_2(w, y): \text{ Cost function} \end{cases}$$

B. Examples

– Perfect complement: $y = \min\{x_1, x_2\}$

$$\rightarrow x_1(w, y) = x_2(w, y) = y$$
$$c(w, y) = w_1x_1(w, y) + w_2x_2(w, y) = (w_1 + w_2)y$$

– Perfect substitutes: $y = x_1 + x_2$

$$\rightarrow x(w, y) = \begin{cases} (y, 0) & \text{if } w_1 < w_2 \\ (0, y) & \text{if } w_2 < w_1 \end{cases}$$
$$c(w, y) = \min\{w_1, w_2\}y$$

– Cobb-Douglas: $y = Ax_1^a x_2^b \rightarrow \begin{cases} TRS = \frac{ax_2}{bx_1} = \frac{w_1}{w_2} \\ y = Ax_1^a x_2^b \end{cases}$

$\rightarrow c(w, y) = Kw_1^{\frac{a}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}$, where K is a constant depending on a, b, and A

$\rightarrow$ Monopolist buys less input than competitive firm does

III. Monopsony

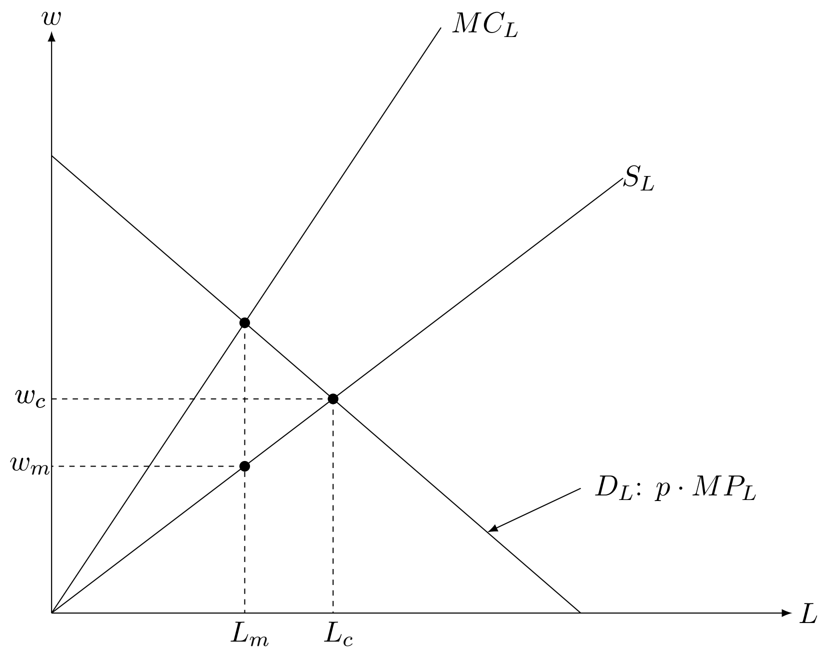
– Monopsonistic input market + Competitive output market

A. Input choice

$$\max_x \; pf(x) - w(x)x$$

$$\xrightarrow{F.O.C.} \underbrace{pf'(x)}_{=MRP} = \underbrace{w'(x)x + w(x)}_{=MC_x} = w(x)\left[1 + \frac{x}{w(x)}\frac{dw(x)}{dx}\right] = w(x)\left[1 + \frac{1}{\eta}\right],$$

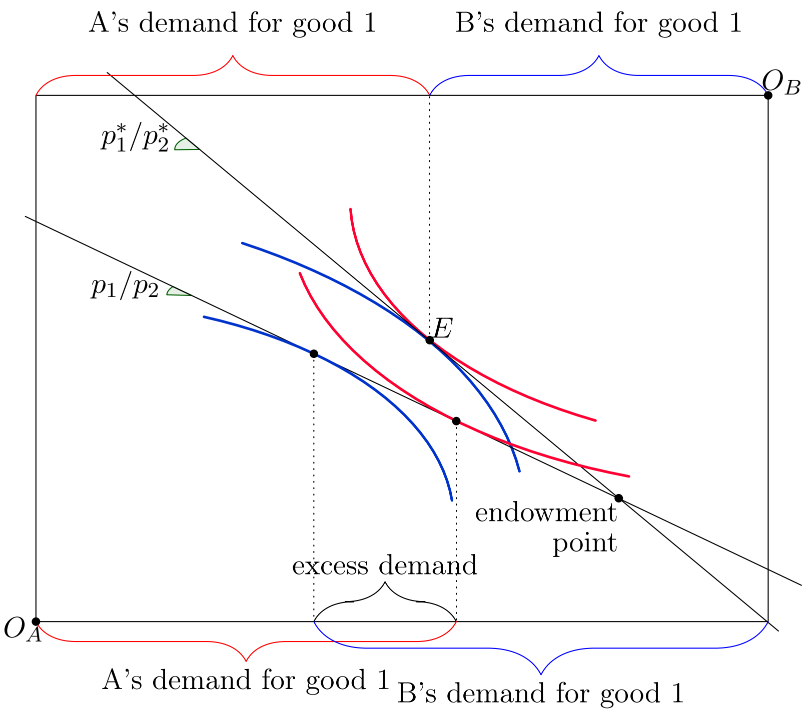
where $\eta \equiv \frac{w}{x}\frac{dx}{dw}$ or the supply elasticity of the factor



Example. $w(x) = a + bx$: (inverse) supply of factor x

$$\rightarrow MC_x = \frac{d}{dx}\left[w(x)x\right] = \frac{d}{dx}\left[ax + bx^2\right] = a + 2bx$$

B. Minimum wage under monopsony



– If (p_1^*, p_2^*) is equilibrium prices, then (tp_1^*, tp_2^*) for any $t > 0$ is equilibrium prices as well so only the relative prices p_1^*/p_2^* can be determined.

– A technical tip: Set $p_2 = 1$ and ask what p_1 must be equal to in equilibrium.

C. Walras' Law

– The value of aggregate excess demand is identically zero, i.e.

$$p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) \equiv 0.$$

– The proof simply follows from adding up two consumers' budget constraints

$$\begin{array}{rl} & p_1 e_A^1(p_1, p_2) + p_2 e_A^2(p_1, p_2) = 0 \\ + & p_1 e_B^1(p_1, p_2) + p_2 e_B^2(p_1, p_2) = 0 \\ \hline & p_1[\underbrace{e_A^1(p_1, p_2) + e_B^1(p_1, p_2)}_{z_1(p_1, p_2)}] + p_2[\underbrace{e_A^2(p_1, p_2) + e_B^2(p_1, p_2)}_{z_2(p_1, p_2)}] = 0 \end{array}$$

– Any prices (p_1^*, p_2^*) that make the demand and supply equal in one market, is guaranteed to do the same in the other market

– Implication: Need to find the prices (p_1^*, p_2^*) that clear one market only, say market 1,

$$z_1(p_1^*, p_2^*) = 0.$$

– In general, if there are markets for n goods, then we only need to find a set of prices that clear $n - 1$ markets.