1. UNITS

Measurements of physical quantities take place by means of a comparison with a standard. For example: a meter stick, a weight of 1 kilogram, etc.

The **base units** that will be used in this course are:

- **meter (m)**: One meter is equal to the path length traveled by light in vacuum during a time interval of 1/299,792,458 of a second.
- **kilogram (kg)**: One kilogram is the mass of a Platinum-Iridium cylinder kept at the International Bureau of Weights and Measures in Paris.
- **second (s)**: One second is the time occupied by 9,192,631,770 vibrations of the light (of a specified wavelength) emitted by a Cesium-133 atom.

A unit that is being used as a base unit must be both accessible and invariable. The original meter bar kept in Paris was not very accessible (this is still true for the kilogram). In addition, the length of the standard bar is temperature dependent. The definition of the meter in terms of the number of wavelengths of a particular atomic transition in Krypton-86 made the meter more accessible; the transition is characteristic for Krypton-86, and is the same for each Krypton-86 atom. However, Doppler shifts due to the thermal motion of the atoms can slightly change the wavelength, and produce a small uncertainty in the definition of the meter. The current definition of the meter in terms of the speed of light is not affected by thermal motion: the speed of light in vacuum is constant, independent of temperature and velocity of observer and/or source.

Example: how to measure the distance d from the earth to the moon ?

APOLLO astronauts placed a mirror on the moon. It can be used to measure the distance between the earth and the moon very accurately. The reflection of a laser beam aimed at this mirror reaches the earth after 2.495 s. The distance can then be calculated:

$$d = \frac{1}{2} \times 2.495 \times 299{,}792{,}458 = 3.74\ 10^{8}\ \mathrm{m}$$

The definition of the standard mass makes it very inaccessible. In principle, the weight of individual nuclei can be used as a standard; nuclear weight does not depend on location, temperature, pressure, etc. However, counting the number of nuclei in a standard (or assembling a fixed number of nuclei) is an almost impossible task.

Note:

1. Improved definitions of base units must be defined such that it matches the previous definition as closely as possible (no need to change all meter sticks in 1983).

2. Speed of light is now defined as 299,792,458 m/s.

All physical quantities are expressed in terms of **base units**. For example, the velocity is usually given in units of m/s. All other units are **derived units** and may be expressed as a combination of base units. For example (see Appendix A):

$$1\ \mathrm{W} = 1\ \frac{\mathrm{J}}{\mathrm{s}} = 1\ \frac{\mathrm{N\ m}}{\mathrm{s}} = 1\ \mathrm{kg}\ \frac{\mathrm{m}^2}{\mathrm{s}^3}$$

Other examples are:

- density: kg/m^3
- area: m^2

$$y(t) = v_o \sin(\theta)\, t - \frac{1}{2} g\, t^2$$

At impact, y(t) = 0. The time of impact can therefore be obtained by requiring that y(t) = 0 and solving for t:

$$y(t) = t\left(v_o \sin(\theta) - \frac{1}{2} g\, t\right) = 0$$

This equation has two solutions:

$$t = 0$$

$$t = \frac{2\, v_0 \sin(\theta)}{g}$$

The first solution corresponds to the time that the projectile was launched, while the second solution gives us the time that the projectile hits the ground again. The x-coordinate at that time can be obtained by substituting the expression for t into the expression for x(t):

$$x = v_0 \cos(\theta) \left(\frac{2\, v_0 \sin(\theta)}{g}\right) = \frac{v_0^{\;2}}{g} \sin(2\theta) = R$$

The maximum range is obtained when sin(2[theta]) = 1, which corresponds to [theta] = 45deg.. The velocity of the projectile on impact can be calculated using the equations for $v_x(t)$ and $v_y(t)$:

$$v_x = v_0 \cos(\theta)$$

$$v_y = v_0 \sin(\theta) - g\, \frac{2\, v_0 \sin(\theta)}{g} = -\, v_0 \sin(\theta)$$

Comparing the velocity on impact with the velocity at t = 0, we observe that the velocity component parallel to the x-axis is unchanged, while the component along the y-axis changed sign.

If we look at the equation of the range R, we observe that the for each value of R (less than R_{max}) there are two possible launch angles: 45deg. + [Delta][theta] and 45deg. - [Delta][theta] (sin(2[theta]) is symmetric around [theta] = 45deg.). The time of flight for the two cases are however different: a larger launch angle corresponds to a longer time of flight (time of flight is proportional to sin([theta])).

Note: in all our calculations we have neglected air resistance.

Sample Problem 4-7

A movie stunt man is to run across a rooftop and then horizontally off it, to land on the roof of the next building (see Figure 4-16 in Halliday, Resnick and Walker). Before he attempts the jump, he wisely asks you to determine whether it is possible. Can he make the jump if his maximum rooftop speed is 4.5 m/s ?

The coordinate system is chosen such that the origin is defined as the position of the stunt man at the moment he starts his jump from the roof (this is also defined as time t = 0). In this case, the following initial conditions apply:

x_0 = 0 m

y_0 = 0 m

The coefficient of static friction can be easily obtained from these two equations:

$$\mu_s = \frac{W\sin(\theta)}{N} = \frac{W\sin(\theta)}{W\cos(\theta)} = \tan(\theta)$$

Note The friction force between car tires and the road is reduced when the car travels uphill or downhill. It is harder to drive uphill or downhill when the roads are slick than it is to drive on leveled surface.

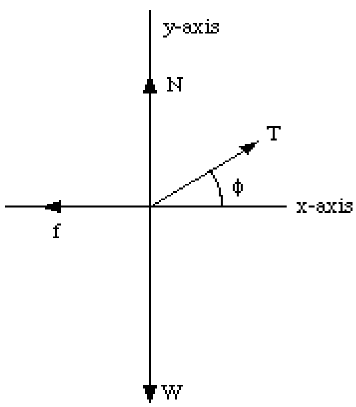


Figure 6.3. Free-Body Diagram for Sled.

Sample Problem 6-3

A woman pulls a loaded sled (mass m) along a horizontal surface at constant speed. The coefficient of kinetic friction between the runners and the snow is u_k and the angle between the rope and the horizontal axis is [phi] (see Figure 6.3). What is the tension in the rope ?

Since the sled is moving with a constant velocity, the net force on the sled must be zero. Decomposing the net force into its components along the x-axis and the y-axis, we obtain the following equations of force:

$$\sum F_x = T\cos(\phi) - f = T\cos(\phi) - \mu_k N = 0$$

$$\sum F_y = N + T\sin(\phi) - W = 0$$

The second equation can be used to eliminate N:

$$N = W - T\sin(\phi)$$

Substituting this expression in the first equation we obtain:

$$T\cos(\phi) - \mu_k\left(W - T\sin(\phi)\right) = 0$$

The tension T can now be calculated:

$$T = \frac{\mu_k W}{\cos(\phi) + \mu_k\sin(\phi)} = \frac{\mu_k m g}{\cos(\phi) + \mu_k\sin(\phi)}$$

The normal force N is always perpendicular to the surface. In the previous two sample problems, the normal force N was proportional to the weight of the object. However, this is not always true. For example, suppose I am pressing an eraser

Sample Problem 8-4

A spring of a spring gun is compressed a distance d from its relaxed state. A ball if mass m is put in the barrel. With what speed will the ball leave the barrel once the gun is fired ?

Suppose E_i is the mechanical energy of the system when the spring is compressed. Since the system is initially at rest, the total energy is just the potential energy of the compressed spring:

$$E_i = U_i + K_i = \frac{1}{2} k d^2$$

At the moment that the ball leaves the barrel, the spring is in its relaxed position, and its potential energy is zero. The total energy at that point is therefore just the kinetic energy of the moving mass:

$$E_f = U_f + K_f = \frac{1}{2} m v^2$$

Conservation of energy requires that $E_i = E_f$. This means

$$\frac{1}{2} k d^2 = \frac{1}{2} m v^2$$

We can now calculate the velocity of the ball

$$v = d\sqrt{\frac{k}{m}}$$

Example Problem 1

Suppose the ball in Figure 8.1 has an initial velocity v_0 and a mass m. If the spring constant is k, what is the maximum compression of the spring ?

In the initial situation, the spring is in its relaxed position (U = 0). The total energy of the ball-spring system is given by

$$E_i = U_i + K_i = \frac{1}{2} m v_0^2$$

The maximum compression of the spring will occur when the ball is at rest. At this point the kinetic energy of the system is zero (K = 0) and the total energy of the system is given by

$$E_f = U_f + K_f = \frac{1}{2} k d^2$$

Conservation of energy tells us that $E_i = E_f$, and thus

$$\frac{1}{2} m v_0^2 = \frac{1}{2} k d^2$$

and

$$m_1 v_{1i} = m_1 v_{1f} \cos\left(\theta_1\right) + m_2 v_{2f} \cos\left(\theta_2\right)$$

and

$$0 = m_1 v_{1f} \sin\left(\theta_1\right) + m_2 v_{2f} \sin\left(\theta_2\right)$$

If the collision is elastic, kinetic energy also needs to be conserved:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

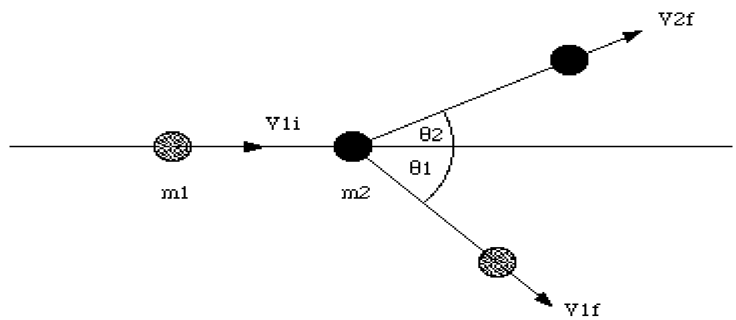


Figure 10.4. A Collision in Two Dimensions.

The variables are:

- mass: m_1 and m_2
- velocity: v_{1i}, v_{1f} and v_{2f}
- angle: [theta]$_1$ and [theta]$_2$

If 4 of these variables are defined, the remaining 3 can be calculated by applying conservation of energy and linear momentum.

Example Problem 10-3

A beam of nuclei with mass m_1 and velocity v_1 is incident on a target nucleus with mass m_2 which us initially at rest. The velocity and scattering angles of both reaction products is measured. Determine the masses of the reaction products and the change in kinetic energy.

The collision is schematically shown in Figure 10.5. Conservation of linear momentum along the x-axis requires

$$p_1 = p_3 \cos(\theta_3) + p_4 \cos(\theta_4)$$

where p_1, p_3 and p_4 are the momenta of particle 1, particle 3 and particle 4, respectively. Conservation of linear momentum along the y-axis requires

$$0 = p_3 \sin(\theta_3) - p_4 \sin(\theta_4)$$

The last equation can be rewritten as

13.1. Equilibrium

An object is in **equilibrium** if the linear momentum of its center of mass is constant and if its angular momentum about its center of mass is constant:

P = constant

L = constant

An object is in **static equilibrium** if its linear momentum and angular momentum is equal to zero:

P = 0 kg m/s

L = 0 kg m^2/s

13.2. Requirements for Equilibrium

If a body is in **translational equilibrium** then dP/dt = 0, or

$$\frac{d\bar{P}}{dt} = \sum \bar{F}_{ext} = 0$$

If a body is in **rotational equilibrium** then dL/dt = 0, or

$$\frac{d\bar{L}}{dt} = \sum \bar{\tau}_{ext} = 0$$

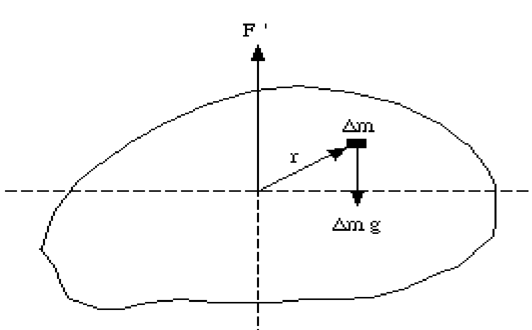
In summary, the following equations must be satisfied for an object in static equilibrium

$$\begin{aligned} &\sum F_x = 0 & & \sum \tau_x = 0 \\ &\sum F_y = 0 & \text{and} \quad & \sum \tau_y = 0 \\ &\sum F_z = 0 & & \sum \tau_z = 0 \end{aligned}$$

If we restrict ourselves to two dimensions (the x-y plane) the following equations must be satisfied:

$$\begin{aligned} &\sum F_x = 0 \\ &\sum F_y = 0 \\ &\sum \tau_z = 0 \end{aligned}$$

13.3. Equilibrium and the Force of Gravity



$$\omega^2 - i\omega \frac{b}{m} - \frac{k}{m} = 0$$

and the solutions for [omega] are

$$\omega = \frac{1}{2}\left(i\frac{b}{m} \pm \sqrt{4\frac{k}{m} - \frac{b^2}{m^2}}\right) \approx \frac{1}{2} i \frac{b}{m} \pm \sqrt{\frac{k}{m}}$$

Substituting this in the expression for x(t) we obtain

$$x(t) \approx x_m \exp\left(-\frac{b\,t}{2\,m}\right) \exp\left(i\,t\sqrt{\frac{k}{m}}\right)$$

We see that the amplitude of the motion gradually decreases over time. This is also true for the kinetic energy of the oscillator. At any point the mechanical energy of the oscillator can be calculated using the expression for x(t):

$$E(t) = \frac{1}{2} k\, x_m{}^2 \exp\left(-\frac{b\,t}{m}\right)$$

Example: Problem 87P

A damped harmonic oscillator involves a block (m = 2 kg), a spring (k = 10 N/m), and a damping force F = - b v. Initially it oscillates with an amplitude of 0.25 m; because of the damping, the amplitude falls to three-fourths of its initial value after four complete cycles. (a) What is the value of b ? (b). How much energy is lost during these four cycles ?

The time dependence of the amplitude of the oscillation is given by

$$A(t) \approx x_m \exp\left(-\frac{b\,t}{2\,m}\right)$$

The period of one oscillation is given by

$$T = \frac{2\,\pi}{\omega}$$

The amplitude after 4 oscillations is therefore given by

$$A\left(t = \frac{8\,\pi}{\omega}\right) \approx x_m \exp\left(-\frac{b}{2\,m}\frac{8\,\pi}{\omega}\right) = \frac{3}{4} x_m$$

The angular frequency [omega] is related to the spring constant k and mass m in the following manner

$$\omega = \sqrt{\frac{k}{m}} = 2.2 \text{ rad/s}$$

Using this expression we obtain for b

$$b = -\frac{2\,m\,\omega}{8\,\pi} \ln\left(\frac{3}{4}\right) = 0.1 \text{ kg/s}$$

The mechanical energy lost during these 4 oscillation can also be easily calculated

The temperature T in this formula must be expressed in Kelvin:

Ti = 293 K

Tf = 308 K

The units for the volume and pressure can be left in l and atm. since only their ratio enter the equation. We conclude that pf = 22 atm.

18.3. Pressure and Temperature: A Molecular View

Let n moles of an ideal gas be confined to a cubical box of volume V. The molecules in the box move in all directions with varying speeds, colliding with each other and with the walls of the box. Figure 18.1 shows a molecule moving in the box. The molecule will collide with the right wall. The result of the collision is a reversal of the direction of the x-component of the momentum of the molecule:

$$p_{ix} = + m\, v_x$$

$$p_{fx} = - m\, v_x$$

The y and z components of the momentum of the molecule are left unchanged. The change in the momentum of the particle is therefore

$$\Delta p = p_{fx} - p_{ix} = -2\, m\, v_x$$

After the molecule is scattered of the right wall, it will collide with the left wall, and finally return to the right wall. The time required to complete this path is given by

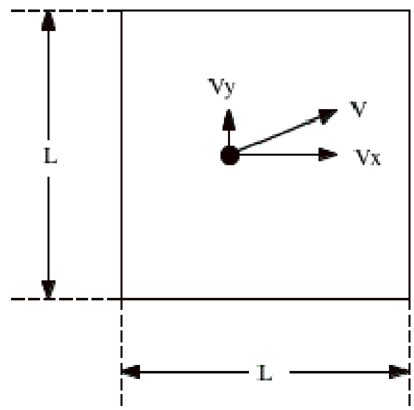


Figure 18.1. Molecule moving in box.

$$\Delta t = \frac{2\, L}{v_x}$$

Each time the molecule collides with the right wall, it will change the momentum of the wall by Δp. The force exerted on the wall by this molecule can be calculated easily

$$F = \frac{\Delta p}{\Delta t} = \frac{2\, m\, v_x}{\left(\frac{2\, L}{v_x}\right)} = \frac{m\, v_x^{\;2}}{L}$$

For n moles of gas, the corresponding force is equal to

$$F = \frac{m}{L}\left(v_{1x}^{\;2} + v_{2x}^{\;2} + v_{3x}^{\;2} + \ldots\ldots\right)$$

The pressure exerted by the gas is equal to the force per unit area, and therefore

$$p = \frac{F}{L^2} = \frac{m}{L^3}\left(v_{1x}^{\;2} + v_{2x}^{\;2} + v_{3x}^{\;2} + \ldots\ldots\right)$$

The term in parenthesis can be rewritten in terms of the average square velocity:

The heat delivered to the cold reservoir is given by

$$\left|Q_C\right| = \left|W_C\right| = n\,R\,T_C \ln\left(\frac{V_c}{V_d}\right)$$

Thus,

$$\frac{\left|Q_H\right|}{\left|Q_C\right|} = \frac{T_H \ln\left(\frac{V_b}{V_a}\right)}{T_C \ln\left(\frac{V_c}{V_d}\right)} = \frac{T_H}{T_C}$$

The efficiency of the cannot engine is therefore given by

$$e = \frac{\left|Q_H\right| - \left|Q_C\right|}{\left|Q_H\right|} = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H}$$

No real engine operating between two specified temperatures can have a greater efficiency than that of a cannot engine operating between the same two temperatures.