CIV ENG Y1 STRUCTURAL MECH SUMMARY

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1 TRUSSES

Idealisation

- Relatively lightweight pin-jointed frames that often can be treated as 2D structures.
- To simplify truss analysis, 3 assumptions are made:
  - Truss members are connect to each other at their ends by frictionless pins.
  - External loads are applied only at the truss' joints.
  - The centroidal axes of the truss members meeting at a joint intersect at the pin connection.
- Thus, truss members only carry axial forces and at each joint, $\sum F_x = 0$ and $\sum F_y = 0$.

Stability & statical determinancy

- Both internal and external stability of the truss contribute to overall stability:
  - External stability: All reactions are not parallel and not concurrent.
  - Internal stability: The arrangement of truss members is some extension of a basic triangle.
- In general, for a truss with m members, j pin joints and r reaction components:
  - If $m + r < 2j$, then the truss is unstable.
  - If $m + r = 2j$, then the truss is statically determinate.
  - If $m + r > 2j$, the the truss is statically indeterminate.

Axial force analysis

Method of joints

1. Find all reactions of the truss.
2. Treating all joints as free bodies, apply $\sum F_x = 0$ and $\sum F_y = 0$.
3. Assume initially that all members are in tension and pull away from the joints.
   - If the force is negative, then the associated member is in compression.
   - Most useful when axial forces in all members are required.

Method of sections

1. Find all reactions of the truss.
2. Split the truss into 2 sections, cutting through at most 3 members.
3. Treating each section as a free body, apply $\sum F_x = 0$, $\sum F_y = 0$ and $\sum M = 0$.
    - Assume initially that there are positive tensile forces in all the cut members.
    - If the force is negative, then the associated member is in compression.
    - Most useful when axial forces in some members are required.

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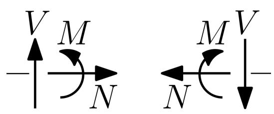
2 BEAMS & FRAMES

Sign convention

In a cut beam, positive shear forces V, axial forces N and bending moments M are oriented as shown:



Shear force & bending moment diagrams

1. Calculate the reaction forces experienced by the beam.
2. Along the beam, determine V and/or M at critical points where loading conditions change.
    - At pin joints and at beam-end supports that allow rotation, $M = 0$.
    - If the beam splits, for each branch, every point just before the junction is a critical point.
    - For the same setup, results for V and M can be superposed for different cases.
3. Use these values to plot the appropriate SFD and BMD.
    - Use the relationships $\frac{dV}{dx} = q$ and $\frac{dM}{dx} = -V$ to predict the shape of the graphs, where q is the force per unit length acting on the beam, x is the distance along the beam from one end.
    - Positive V and negative M regions lie above the beam.
4. Check these diagrams against the deflected shape of the beam.

Useful results

Loading type	$\boldsymbol{q}$	$\boldsymbol{V = \int q\ dx}$	$\boldsymbol{M = \int -V\ dx}$
Point load	–	k	$-kx + A$
Uniformly distributed	k	$kx + A$	$kx^2 + Ax + B$
Linearly distributed	$kx + A$	$kx^2 + Ax + B$	$kx^3 + Ax^2 + Bx + C$

3 STRESS & STRAIN

Definitions

Stress, σ: The force F acting on an element divided by the area A that it acts over.

$$\sigma = \frac{F}{A}$$

Strain, ε: The deformation δL divided by the original length L of the element.

$$\varepsilon = \frac{\delta L}{L}$$

Young's modulus, E: The ratio of stress σ to strain ε in the linear-elastic range of a material.

$$E = \frac{\sigma}{\varepsilon} = \frac{FL}{A\delta L}$$

Stress-strain curves

- Materials usually show a linear-elastic region, where stress is directly proportional to strain.
- Above the yield stress, plastic behaviour is observed, where strains are large under approximately constant loads and irreversible.
- Eventually, the material fractures when it reaches its ultimate load.

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4 BENDING STRESSES

Definitions

Beam: A structural member that sustains perpendicular loads mainly through its bending moment.
Sagging: When the beam's bottom is under tension and its top under compression.
Hogging: When the beam's top is under tension and its bottom under compression.

Moment of resistance

- Assumptions:
  - When bending, the beam's cross section remains symmetrical about at least 1 of the y- or z-axes.
  - Materials are linear-elastic, isotropic and planar sections remain planar.
- When bending, each beam fibre's deformation, and thus, strain and stress vary linearly with its distance from the neutral axis.