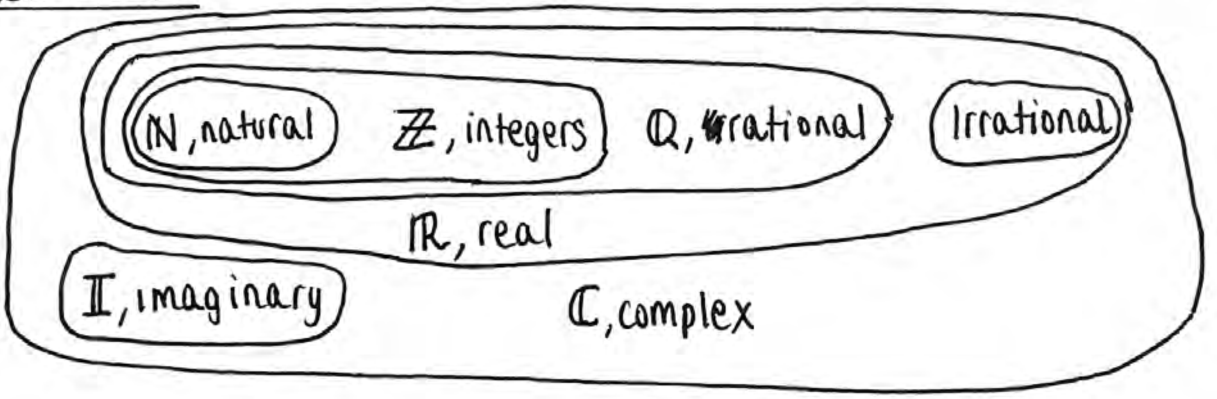
PURE

1) ALGEBRAIC METHODS

1.1 - Proof by contradiction   ⁘ → contradiction

To prove a statement by contradiction you start by assuming it is not true. You then use logical steps to show that this assumption leads to something impossible (either a contradiction of the assumption or a contradiction of a fact you know to be true). You can then conclude that your assumption was incorrect, and that the original statement was true



* Closed set - a set is closed (under an operation) if and only if the operation of any two elements of the set produces another element of the same set. If the operation produces even one element outside the same set, the operation is not closed.

- $\mathbb{N}$ → closed under addition and multiplication ['−' gives neg $2-3=-1$ / '÷' gives $\mathbb{Q}$ $\frac{2}{3}$]
- $\mathbb{Z}$ → closed under addition, multiplication and substraction ['÷' gives $\mathbb{Q}$ $-\frac{2}{3}$]
- $\mathbb{Q}$ → closed under addition, multiplication, substraction and division (if it is not by 0)
- '$\mathbb{Q}$ → not closed under any operation. (irrational)

⤳ If properties don't apply to your assumption you can use it is a ⁘.

A rational number can be written as $\frac{a}{b}$, where 'a' and 'b' are integers.
(cannot be further reduced)

An irrational number cannot be expressed in the form $\frac{a}{b}$, where 'a' and 'b' are integers.

List of proofs

- Prove that the set $\mathbb{N}$ is unbounded above
- Prove that there is no higher prime number
- Prove by contradiction that if n^2 is even the n must be even
- Prove $\sqrt{2}$ irrational
- Prove by contradiction that if n^2 is a multiple of 3, n is a multiple of 3.
- Prove $\sqrt{3}$ irrational (any $\sqrt{\text{int}}$ '$\mathbb{Q}$)

1.2- Algebraic fractions

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$

máximo común divisor (mcd) / highest common factor (hcf):

- comunes al menor exponente.

mínimo comun múltiplo (mcm) / Lowest common multiple (Lcm):

- comunes y no comunes al mayor exponente.

1.3) Partial fractions

⇒ numerator < denominator ⇐

A single fraction with two distinct linear factors can be split into two separate fractions with linear denominators.

quadratic denominator: $\frac{\text{Linear}}{(ax+b)(cx+d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$

also with three distinct linear factors

cubic denominator: $\frac{\text{Linear / quadratic}}{(ax+b)(cx+d)(ex+f)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(ex+f)}$

1.4) Repeated factors

cubic ~~quadratic~~ denominator: $\frac{\text{Linear / quadratic}}{(ax+b)^2(cx+d)} = \frac{A}{(ax+b)} + \frac{B}{(ax+b)^2} + \frac{C}{(cx+d)}$

also with repeated linear factor

~~use roots~~ - once in form Linear/quadratic = $A(cx+d) + B(ax+b)$

1) use roots $(x-2) \rightarrow x=2$ $(3x-2) \rightarrow x = 2/3$ or $(x^2+x)/x \rightarrow x=0$

2) substitute values found (with one letter left) → chose a value for x (x=1) and find letter left.

3) if two letters left → chose a value for x → simultaneous equation.

1.5 - Algebraic division

($N^x \geq D^x$)

Improper fraction → numerator has a degree equal or higher than the denominator
proper fraction → denominator's order > numerators

An improper fraction must be converted to a mixed fraction before you can express it in partial fraction

improper fraction → mixed fraction → partial fraction

- degree top = degree bottom ~~AB~~ $A + \frac{B}{(\ )} + \frac{C}{(\ )} \dots$
- degree top = degree bottom +1 $Ax + B + \frac{C}{(\ )} + \frac{D}{(\ )} \dots$
- degree top = degree bottom +2 $Ax^2 + Bx + C + \frac{D}{(\ )} \dots$

To convert an improper fraction into a mixed you can use: algebraic division or the relationship $F(x) = Q(x) \times$ divisor + reminder (and then compare coefficients)

1) Algebraic division : $\frac{F(x)}{\text{divisor}} \equiv Q(x) + \frac{\text{remainder}}{\text{divisor}}$

2) multiply by divisor and compare coefficients : $F(x) \equiv Q(x) \times$ divisor + remainder

2) FUNCTIONS AND GRAPHS

2.1 - The modulus functions

modulus of a function = absolute value function (Abs)

A modulus function is, in general, a function of the type $y = |F(x)|$
- when $F(x) \geq 0$, $|F(x)| = F(x)$
- when $F(x) < 0$, $|F(x)| = -F(x)$

- Solve modulus:
1) draw graph
2) ± on modulus side
→ You might be asked to show a inequality
→ 2 equations

To sketch the graph of $y = |ax+b|$, sketch $y = ax+b$, then reflect the section of the graph below the x-axis in the x-axis