FURTHER STATISTICS

1) DISCRETE RANDOM VARIABLES

1.1- Expected value of a discrete random variable

Random variable → variable whose value depends on a random event.

The random variable is discrete if it can only take certain numerical values.

If you take a set of observations from a discrete random variable, you can find the mean of those observations.

↳ As the number of observations increases, this value will get closer and closer to the expected value of the discrete random variable.

The expected value of the discrete random variable X is denoted $E(X)$ and defined as:

$$E(X) = \sum x \, P(X=x)$$

theoretical value.

The expected value is sometimes referred to as the mean, and is denoted by μ.

Also,

$$E(X) = \sum x^2 P(X=x)$$

$[P(X^2 = x^2) = P(X=x)]$

Use $\sum P(X=x) = 1$ to solve simultaneously.

1.2- Variance of a discrete random variable

The variance of X is usually written as $Var(X)$ and is defined as:

$$Var(X) = E((X-E(X))^2)$$

Sometimes it is easier to calculate the variance using the formula

$$Var(X) = E(X^2) - (E(X))^2$$

$(Var(X)$

1.3- Expected value and variance of a function of X

$$E(g(X)) = \sum g(x) P(X=x)$$

- If X is a random variable and a and b are constants, then:

$$E(aX+b) = aE(X)+b$$

- If X and Y are random variables, then:

$$E(X+Y) = E(X)+E(Y)$$

$Var(X+Y) = Var(X)+Var(Y)$

- If X is a random variable and a and b are constants then:

$$Var(aX+b) = a^2 Var(X)$$

1.4 - Solving problems involving random variables

Suppose you have two random variables X and $Y = g(X)$. If g is one-to-one, and you know the mean and variance of Y, then it is possible to deduce the mean and variance of X.

2) POISSON DISTRIBUTIONS

2.1 - The poisson distribution

If $X \sim Po(\lambda)$, then the Poisson distribution is given by

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, 2, 3 \ldots$$

This is an infinite probability distribution $P(X=x)>0$ for any positive integer X, although as X gets large, the probabilities get very small.

You can use the Poisson cumulative distribution function on your calculator to find $P(X \leq x)$ for other values of λ and x.

STATISTICS $\boxed{2}$ → DIST $\boxed{F5}$ → D $\boxed{F6}$ → Poisson $\boxed{F1}$

Ppd (exact value) $\boxed{F1}$

Pcd (cumulative) $\boxed{F2}$

InvP (inverse poisson) $\boxed{F3}$

- adjust mean to interval!

2.2 - Modelling with the Poisson distribution

In order for the Poisson distribution to be a good model, the events must occur:

- independently
- singly, in space or time
- at a constant rate so that the mean number in an interval is proportional to the length of the interval.

The parameter λ, in the Poisson distribution is the average number of times that the event will occur in a single interval.

2.3 - Adding Poisson distributions

If $X \sim Po(\lambda)$ and $Y \sim Po(\mu)$, then $X+Y \sim Po(\lambda+\mu)$

2.4 - Mean and variance of a Poisson distribution

If $X \sim Po(\lambda)$

- Mean of X = $E(X) = \lambda$
- Variance of X = $Var(X) = \sigma^2 = \lambda$

The mean is equal to the variance in the Poisson distribution.
↓
This is a useful indicator of whether or not a Poisson distribution is a suitable model for a particular distribution.

2.5 - Mean and variance of the binomial distribution

If $X \sim B(n, p)$

- Mean of X = $E(X) = \mu = np$
- Variance of X = $Var(X) = \sigma^2 = np(1-p)$

Remember → $P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}$

So if $P(X=0) = (1-p)^n$

2.6 - Using the Poisson distribution to approximate the binomial distribution

If $X \sim B(n, p)$ and: n is large and p is small, then X can be approximated by $Po(\lambda)$, where $\lambda = np$.

$(1-p)$ close to 1 if p is small so $Var(X) = np(1-p) = np = \lambda$