Advanced Mechanics
Projectile Motion
Projectiles and Acceleration
- An object launched with an initial velocity at an initial angle, not acted upon by forces during flight
- Gravity is constant for HSC ($g = 9.8ms^{-2}$)
- Air resistance is negligible for HSC
- Usually separated into vertical and horizontal components
    - $v_x = u\cos\theta$
    - $v_y = u\sin\theta + gt$

Horizontal and vertical direction and velocity
- Horizontal and vertical motion are independent
- Horizontal
    - $v_x = u_x$ horizontal velocity is the same as initial until landing
    - $s_x = u_x t$
- Vertical
    - $v_y = u_y + a_y t$
    - $s_y = u_y t + \frac{1}{2}a_y t^2$
    - ${v_y}^2 = {u_y}^2 + 2a_y s_y$ useful for vertical velocity at a specific position rather than time

Circular Motion
Uniform circular motion
- Uniform circular motion is when an object moves in a circle at a constant (uniform) speed
- Constant speed, but there is acceleration as there is a change in direction, therefore changing velocity
- Force causing this movement is centripetal force
    - Can come from a variety of sources: gravity (orbits), friction (car turning) or tension (spinning string)

Centripetal forces
- Standard examples
    - Car moving around a bend: Force created by the friction of the tires and the road
    - Mass on a string: Force created by the tension in the string
    - Objects on banked tracks: Gravity acting downward, then a normal force perpendicular to the track. Centripetal comes from the vector sum

Quantitative predictions and formulas
- $F_c = \frac{mv^2}{r}$
- $a_c = \frac{v^2}{r}$
- $v = \frac{2\pi r}{T}$
- $\omega = \frac{\Delta\theta}{\Delta t}$

Energy and work
- Energy is constant in uniform circular motion
- $E_k = \frac{1}{2}mv^2$
- No change in kinetic energy means no work being done

Torque as a rotational force
- Torque: The turning moment of a force, rotational force
- $T = r_{\perp}F = rFsin\theta$
    - Angle between the radius and the force applied

Motion in Gravitational Fields

Gravitational fields and Newton's Law

- Gravitational field surrounds any object with mass and theoretically infinite in size
- The heavier and denser the object the stronger the field
- Moving against a gravitational field requires work to be done and gives gravitational potential energy to the object being moved
- Newton's Law of Universal Gravitation allows it to be determined the force of attraction between two bodies due to their gravity
    - $F = \frac{GMm}{r^2}$
        - Usually M is the bigger object and m is the smaller one
        - G is the Universal Gravitational Constant ($6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$)
    - Can be linked to Newton's 2nd law to find strength of field at a certain point
    - $g = \frac{GM}{r^2}$

Orbital motion

- An example of uniform circular motion where gravity creates the centripetal force
- More gravitational force required to keep a larger mass in orbit
- Acc. due to gravity at a point = centripetal acc. at the same point
- Quantities
    - Orbital radius – distance from the centre of the orbit to the object
    - Orbital velocity – speed the object is moving in its orbit
    - Orbital period – how long to complete an orbit
    - All interrelated
- Types of orbits
    - Low Earth Orbits (LEOs)
        - Orbits with very small orbital radius, less than 8500km, usually above 6800km)
        - Period of about 1.5-2 hours
        - Usually used for surveillance
    - Middle Earth orbits
        - In between LEOs and geostationary (8500km to 42500km)
        - Used for navigation and communication
    - Geostationary orbits
        - Specific orbital period of 24 hours, stay at the same point above the planet at all times
        - Roughly 42500km orbital radius
        - Used for communication and broadcasting

Kepler's Laws

- All orbiting bodies orbit in an elliptical fashion with the body being orbited at one focus of the ellipse, circular motion is assumed for simplicity
- A line joining an orbiting body to the centre of its orbit will sweep out equal area in equal time
- $\frac{r^3}{T^2} = K$
- $F_g = F_c$ therefore through substituting formulas
- $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

Gravitational potential energy

- Energy possessed by an object when it is raised against a gravitational field
- $E_p = mg\Delta h$
- For objects in orbit the zero point occurs at an infinite distance

- $E_{gp} = -\frac{GMm}{2r}$

Escape velocity

- The velocity that would need to be travelled to escape the gravitational field of that planet
- $v_e = \sqrt{\frac{2GM}{r}}$

Electromagnetism

Charged Particles, Conductors, and Electric and Magnetic Fields

Charged particles and uniform electric fields

- Two charged platers with a voltage applied between them will cause an accumulation of charge on either plate
- Plate connected the positive end of the battery will accumulate positive charges and same for negative
- This will create an electric field between the plates, from the positive to the negative
- Size is $E = \frac{V}{d}$
- $E = \frac{F}{q} = \frac{W}{qd} = \frac{V}{d}$
- $V = \frac{W}{q}$
- $W = Fd, W = Eqd, W = qV$
- $F = Eq$ --> $Eq = ma$ --> $a = \frac{Eq}{m}$

Particle trajectories

- Charges can move like projectiles, force comes from the electric field
- Parabolic motion, same formulas can be used

Interactions of charged particles and uniform magnetic fields

- For a charged particle to interact with a magnetic field it must be moving, all moving charges are surrounded by a magnetic field
- Magnetic fields interact with other fields which means the moving particle will interact with magnetic fields
- Magnetic field's force direction is perpendicular to the field rather than parallel, like electric fields
- Right hand rule
  - Fingers in line with magnetic field lines
  - Thumb with direction of motion
  - Palm goes in the direction of the force
  - This is for positively charged, negative use left hand
- $F = qv_{\perp}B = Bqv\sin\theta$
  - Force is largest when field and motion are perpendicular (sin90 = 1)
  - As they move towards parallel force gets smaller
- Force of both electric and magnetic is based on size of field and magnitude of charge
- As electric is projectile, magnetic field motion is uniform circular motion

The Motor Effect

Current-carrying conductors and uniform magnetic fields

- Direction of magnetic field around a current-carrying wire is determined using right-hand grip rule
  - Right hand in thumbs up
  - Thumb goes in direction of conventional current flow (positive to negative)
  - Fingers wrap around the wire, this is the direction of the magnetic field
- If an external magnetic field surrounds the wire then they will interact and put a force on the wire
- Magnitude of the force on the wire: $F = LI_{\perp}B = BIL\sin\theta$
  - B is size of field, I is size of the current, L is length of wire, θ is angle between current and magnetic field lines
  - Force is largest when field and motion are perpendicular (sin90 = 1)
  - As they move towards parallel force gets smaller
  - Direction can be determined using right hand slap