INVESTMENT

WEEK1: Interest Rates and Bonds Valuation

Bonds	
$$\begin{cases} TreasuryBonds(CommonwealthGovernment)\begin{cases} Fixed\ Coupon \\ Face\ Value = \$1000 \\ Semiannual\ coupon \end{cases} \\ CorporateBonds(Firms)\begin{cases} Rated\ by\ bond\ rating\ agencies \\ Callable, Convertable, Floating\ Rate\ Bonds\ldots existed \end{cases} \end{cases}$$	
Zero Coupon Bonds	*Bonds with no coupon payment. Any Bonds can be treated as a portfolio of zeros as long as the discount rate and discounting period matched.*
Callable Bonds	*Bonds that may be repurchased by the issuer at a specified call price during the call period.* *Holders of called bonds forfeit their bonds for the call price, thereby giving up the prospect of an attractive rate of interest on their original investment. To compensate investors for this risk, callable bonds are issued with higher coupons and promised yields to maturity than non-callable bonds.*
Convertible Bonds	*Bonds with an option allowing the bondholder to exchange the bond for a specific number of shares of common stock on the firm.* *Convertible bondholders benefit from price appreciation of the company's stock. This benefit comes at a price that convertible bonds often pay lower coupon rates and promised yield to maturity.*
Put Bonds	*Bonds that the holders may choose to either exchange for par value at some date or to extend for a given number of years.* *It is optimal to extend the term of the bonds when coupon rate exceeds the current market yields.*
Floating Rate Bonds	*Bonds with <u>coupon rates</u> periodically reset according to market rate.* *The major risk involved in floaters for issuers is that the coupon rate on floaters adjusts to changes in the general level of market interest rates but not necessarily coincide with the financial condition of the firm.*
Bond pricing	$$Price = coupon \times \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right] + Par\ Value \times \frac{1}{(1+r)^T}$$
Between coupon dates	$$InvoicePrice = FlatPrice + AccruedInterest\ (simple\ interest)$$ $$\approx Original\ price\ \times (1+r)^t\ (compound\ interest)$$ $$Accrued\ Interest = \frac{days\ since\ last\ coupon\ payment}{days\ between\ two\ coupon\ dates} \times coupon$$
	There should be some return to compensate that the bondholder has held the bond for a period and hasn't received anything. Thus, when selling a bond between two coupon dates, the sellers will receive an accrued interest on top of the flat price to compensate their time value put into the bond.
Bond pricing in EXCEL	$= PRICE(DATE(2019,8,1), DATE(2022,8,1), Coupon\ Rate, Yield\ To\ Matur$ $Par\ Value, Frequency\ Of\ Coupon\ Payment\ Per\ Annum)$ **note that: If the face value is 100, the enter the price and face value as the question stated; however, if the face value is $1,000, then enter 100 and divide the price by 10*

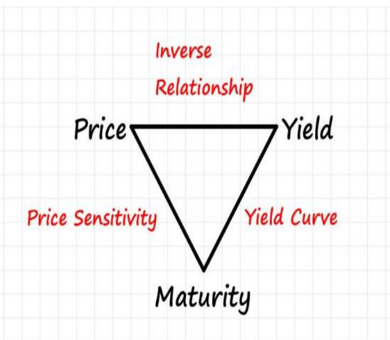
Interest Rates & Bond Yields		
Yield To Maturity	$P_t = \sum_{i=1}^{T} \frac{CF_{t+i}}{(1+r_{YTM})^i}$ $\qquad r_{YTM} = \left(\frac{CF_{t+T}}{P_t}\right)^{\frac{1}{T}} - 1$	
	The yield to maturity is defined as the discount rate that makes the present value of a bond's payments equal to its price. This rate is often viewed as a measure of the average rate of return that will be earned on a bond if it is bought now and held until maturity. *YTM=HPR when 1). Held to maturity. 2) Reinvest coupons to this yield.* *Discount Rate →Price→ YTM*	
Bond Equivalent Yield (BEY)	*An annual discount rate (nominal rate) Also called "Annual Percentage Rate" (APR)*	$r_{BEY} = r_{period} \times \frac{365}{n}\ (Australian)$ $r_{BEY} = r_{period} \times \frac{360}{n}\ (American)$ $r_{BEY} = r_{period} \times n\ periods\ a\ year$
Effective Annual Rate (EAR)	*Accounts for annual compound rate (real rate), also called effective annual yield or annual percentage yield.*	$r_{EAR} = \left(1 + \frac{r_{BEY}}{n}\right)^n - 1$
Current Yield & Coupon Rate	$CY = \frac{Annual\ Coupon}{Bond\ Price} \quad CR = \frac{Annual\ Coupon}{Par}$	
Flat term structure	*when rates of return for all periods are the same (only an assumption, not true in reality. In reality, long term debts tend to have higher rate of return as they are riskier.*	

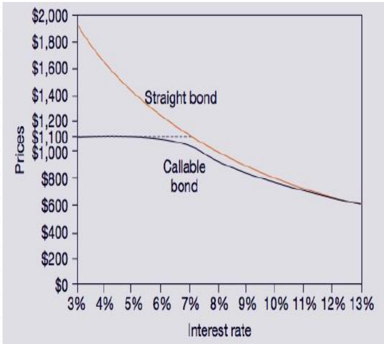
YTM, CR and CY

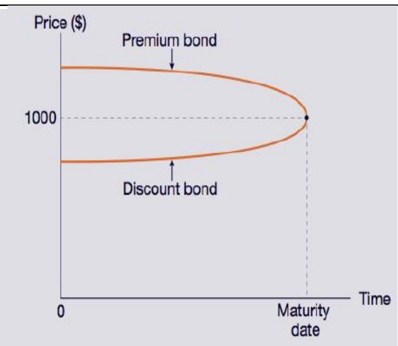
YTM captures all future CFs vs price while CY captures first CF vs price
YTM is (some sort of) weighted average return of each future CFs. Par is of bigger weight than coupons, so bigger impact on YTM.

$$\begin{cases} YTM < CY < CR\ when\ Price > Par\ (YTM\ contains\ capital\ loss\ getting\ from\ price\ to\ par) \\ YTM = CY = CR\ when\ Price = Par \\ YTM > CY > CR\ when\ Price < Par\ (YTM\ contains\ capital\ gain\ getting\ from\ price\ to\ par) \end{cases}$$

Price, Yield & Maturity





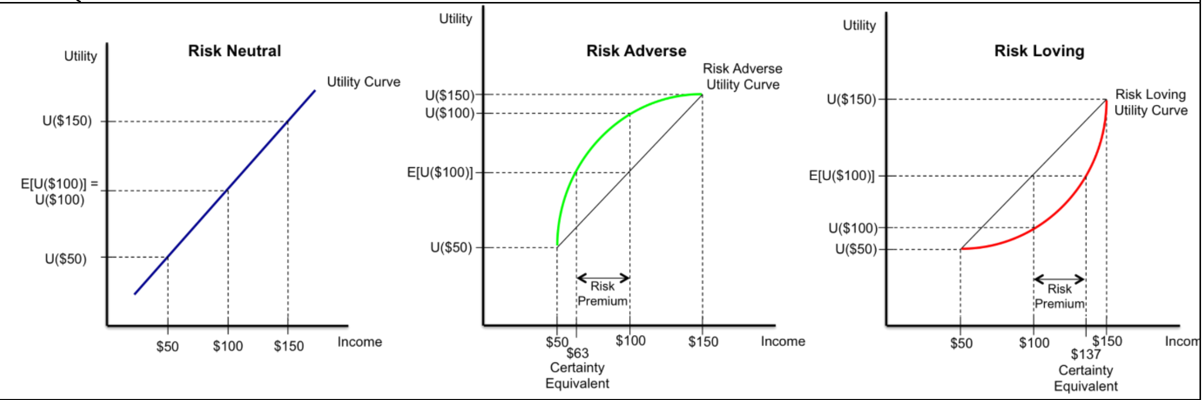


<u>*Note: In bond market, we quote YTM using the BEY method that is, YTM itself is a simple interest rate. It is different to EAY.*</u>
If you are given a yield to maturity for a semi-annual coupon bond, you should treat the period_YTM as annual_YTM/2. If you have a quarterly coupon bond you should treat the period_YTM as annual_YTM/4.
To state this another way, the PERIODIC YTM is in fact the return we receive if we hold the bond to maturity and we can reinvest the returns at the same rate; but, the question is how we annualize, do we use the BEY method or the EAR method. In bond markets, typical is to use the BEY method, which takes the periodic_YTM and multiplies by the number of periods. Please note that for Australian government debt the annualization is yet again slightly different.

WEEK2: Valuing Risky Bonds

Attitudes Towards Risks

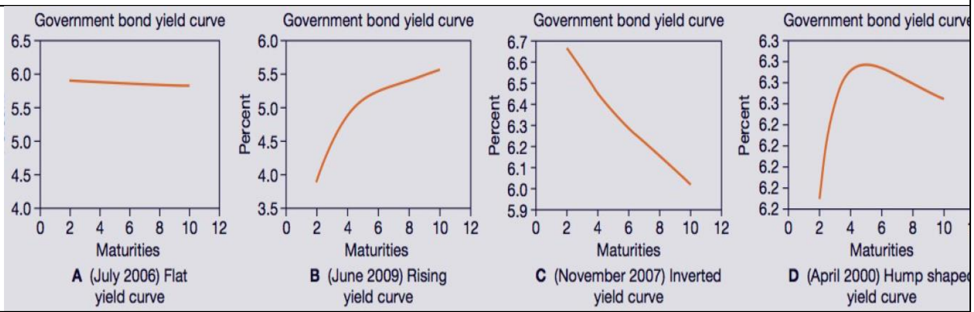
$$\begin{cases} Risk\ Averse: Conpensations\ are\ needed\ for\ them\ to\ take\ risks. \\ Risk\ Neutral: Aren't\ afraid\ of\ risks\ as\ allong\ as\ the\ investment\ is\ a\ fair\ bet. \\ Risk\ Loving: Higher\ Risks\ higher\ return\ is\ better\ than\ lower\ risks\ lower\ return \end{cases}$$



In addition: Risk neutral investors are willing to take risk as long as it's a fair bet. However, they need a fair rate of return that is the rate of return of a risky investment should be the same as a non-risky investment after taking the risk into calculation. Otherwise, you will pick the high return non-risky investment over the risky one.

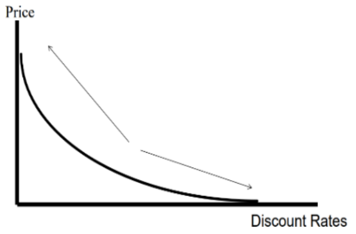
Decomposition of yield

$$\begin{cases} If\ we\ assume\ no\ risk: YTM = Time\ Premium + \cancel{Risk\ Premium + Default\ Premium} \\ If\ we\ allow\ risk\ but\ risk\ neutral: YTM = Time\ Premium + \cancel{Risk\ Premium} + Default\ Prem \end{cases}$$

Expected Cash Flow	$E[\widetilde{CF}_A] = \sum_{s=1}^{S} p(s) * CF_A(s) \quad P = \sum_{i=1}^{\infty} \frac{E[\widetilde{CF}_{t+i}]}{(1+E[r])^\wedge i}$
Expected Return	$E[\tilde{r}_A] = \sum_{s=1}^{S} p(s) * r_A(s) \quad = Time\ Premium + risk\ Premium$
	• *Quoted yield ≠ Expected return (Expected return takes risks into account, quoted yield doesn't)* • *The default premium is acquired by both risk neutral and risk adverse investors. However, risk premium is only acquired by risk averse investors to make them willing to participate.* • *Recovery Rate : How much you can get back if the company defaults. There are people out there who buy the bond after its default at a discount.*
Yield Curve	
Expectation Theory	• *The theory that yields to maturity are determined solely by expectations of future short-term interest rates.* • *Relatively high yields on long-term bonds reflect expectations of future increases in rates, while relatively low yields on long-term bonds (a*

	downward-sloping or inverted yield curve) reflect expectations of falling short-term rates. • *Advocates of the expectations hypothesis commonly invert this analysis to infer the market's expectation of future short-term rates.*
	$(1+y_n)\hat{}n = (1+y_{n-1})\hat{}(n-1) \times (1+f_n)$
The Liquidity Preference Theory	• *Investors in long-term bonds might require a risk premium to compensate them for this risk. In this case, the yield curve will be upward-sloping even in the absence of any expectations of future increases in rates.* • *Issuers of bonds seem to prefer to issue long-term bonds. This allows them to lock in an interest rate on their borrowing for long periods and thus they may be willing to pay higher yields on these issues.*
Market Segmentation and Preferred Habitat Theory	• *Different investors also have different needs for maturities and that may restrict them to invest in bonds in specific maturity segments maturities. Yields in each maturity segment would be governed by demand and supply of securities within that segment, which in turn would affect the shape of the yield curve.*

WEEK3: Bond Portfolio Management

Interest Rate Risk	• *Change in the credit quality of the issuer -* $Risk \downarrow Price \uparrow YTM\ (promised) \downarrow$ • *Change in the yield on comparable bonds – otherwise there would be arbitrage.*		
Interest Rate Sensitivity/ Duration	• *Duration is positively related to maturity* • *Duration is negatively related to coupon rate* • *Duration is negatively related yield to maturity*		
Convexity	*The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increases. In other words, interest rate risk is less than proportional to bond maturity* *(Investors like convexity because more convex bonds. Increase in price more when yields drop than they decrease in price when yields rise.)*	 	
	Formula	$$convexity = \frac{1}{P \times (1+y)^2}\sum_{t=1}^{n}\left[\frac{CF_t}{(1+y)^t}(t^2+t)\right]$$	
Duration	• *Macaiday's duration equals the weighted average of the times to each coupon or principal payment made by the bond. It captures both cash flow and maturity.* • *Duration allows us to quantify sensitivity which greatly enhances our ability to formulate investment strategies.* ❖ *Summary measure of length or effective maturity for a portfolio.* ❖ *Measure of price sensitivity for changes in the interest rate.* ❖ *Immunization of interest rate risk.*		
	Weight of each CF	$$w_t = \frac{CF_t/(1+y)^t}{bond\ price}$$	
	Internal Duration	$$D = \sum_{t=1}^{T} t \times w_t$$	$$\frac{\Delta P}{P} = -D \times \left[\frac{\Delta(1+y)}{1+y}\right]$$
	Modified Duration (dollar duration)	$$D* = \frac{D}{(1+y)}$$	$$\frac{\Delta P}{P} = -D * \times \Delta y$$
	Duration of perpetuity	$$\frac{1+y}{y}$$	
	Duration Cheat	$$D = \frac{1+y}{y} - \frac{1+y+T(c-y)}{c[(1+y)^T-1]+y}$$	
	Sensitivity and convexity	$$\frac{\Delta P}{P} = -D^*\Delta y + \frac{1}{2} \times convexity \times (\Delta y)^2$$	
	• *The duration of zero-coupon bond equals to its time to maturity.* • *The duration of a floating rate bond is zero. (IN REALITY: the actual duration of floating bond is the next payment date as the coupon rate wouldn't be up to date until the next payment date. Because the coupon rate is fixed for a quarter or half year or at most a year, not instantaneous).*		
Immunization	• *Why we need to immunize?* ❖ *Funds should match the interest rate exposure of assets and liabilities so that the value of assets will track the value of liabilities whether rates rise or fall.*		