Topics:

- Ethical and professional standards: 15% **(pg. 2 – 10)**
- Quantitative methods: 12% **(pg. 11 – 58)**
- Economics: 10% **(pg. 59 – 129)**
- Financial reporting and analysis: 20% **(pg. 130 – 194)**
- Corporate finance: 8% **(pg. 195 – 216)**
- Portfolio management: 5% **(pg. 216 – 236)**
- Equity investments: 10% **(pg. 237 – 268)**
- Fixed income investments: 12% **(pg. 269 – 299)**
- Derivatives: 5% **(pg. 300 – 323)**
- Alternative investments: 3% **(pg. 324 – 331)**

Question Make-up

- Economics: 15% of 240 = **36**
- Quantitative Methods: 12% of 240 = **29**
- Economics: 10% of 240 = **24**
- Financial reporting and analysis: 20% of 240 = **48**
- Corporate finance: 8% of 240 = **19**
- Portfolio management: 5% of 240 = **12**
- Equity investments: 10% of 240 = **24**
- Fixed income investments: 12% of 240 = **29**
- Derivatives: 5% of 240 = **12**
- Alternative investments: 3% of 240 = **7**

ETHICAL AND PROFESSIONAL STANDARDS

LOS 1.a: Describe the structure of the CFA Institute Professional Conduct Program and the process for the enforcement of the Code and Standards

The CFA Institute Professional Conduct Program is covered by the CFA

Institute Bylaws and the Rules of Procedure for Proceedings Related to Professional Conduct. The Program is based on the principles of fairness of the process to members and candidates and maintaining the confidentiality of the proceedings. The Disciplinary Review Committee of the CFA Institute Board of Governors has overall responsibility for the Professional Conduct Program and enforcement of the Code and Standards.

The CFA Institute Designated Officer, through the Professional Conduct staff, conducts inquiries related to professional conduct. Several circumstances prompt inquiry:

- Self-disclosure by members/candidates on annual PCS of involvement in civil litigation, criminal investigation or that they are the subject of a written complaint
- Written complaints about a member/candidate's professional conduct
- Evidence of misconduct by a member/candidate received by the PC staff through public sources
- Exam violation

Inquiry process:

- Once an inquiry has begun, the PC staff may request a written explanation from the subject member/candidate and then may:
    - Interview the subject
    - Interview the complainant/third party
    - Collect evidence themselves
- The Designated Officer may decide:
    - No disciplinary action is necessary
    - Issue a cautionary letter
    - Disciplinary action is necessary
        - Here the member/candidate may accept or reject the sanction

LOS 1.b: State the six components of the Code of Ethics and the seven Standards of Professional Conduct

[(Also, LOS 2.a: Demonstrate the application of the Code of Ethics and Standards of Professional Conduct to situations involving issues of professional integrity.

LOS 2.b: Distinguish between conduct that conforms to the Code and Standards and conduct that violates the Code and Standards.

LOS 2.c: Recommend practices and procedures designed to prevent violations of the Code of Ethics and Standards of Professional Conduct.)]

<u>Code of Ethics:</u>

- Act with integrity, competence, diligence, respect, and in an ethical manner with the public, clients, prospective clients, employers, colleagues

in the investment profession, and other participants in the global capital markets
- Place the integrity of the investment profession and the interests of clients above their own personal interests`
- Use reasonable care and exercise independent professional judgment when conducting investment analysis, making investment recommendations, taking investment actions, and engaging in other professional activities
- Practice and encourage others to practice in a professional and ethical manner that will reflect credit on themselves and the profession
- Promote the integrity of, and uphold the rules governing, capital markets
- Maintain and improve their professional competence and strive to maintain and improve the competence of other investment professionals

<u>The Standards of Professional Conduct:</u>

I: Professionalism

A: Knowledge of the law

B: Independence and objectivity

C: Misrepresentation

D: Misconduct

II: Integrity of Capital Markets

A: Material nonpublic information

B: Market manipulation

III: Duties to Clients

A: Loyalty, prudence and care

- Members must seek best price and execution

B: Fair dealing

- Members must not discriminate b/w clients

C: Suitability

- 1. When members/candidates are advising a client they must:
  - a: Make a reasonable enquiry into a client's investment experience, risk/return objectives and financial constraints
  - b: Determine that an investment suits their client's financial situation and written objectives/constraints
  - c: Judge the suitability of investments in the context of the client's total portfolio

- 2. When members/candidates are responsible for managing a portfolio to a specific mandate, they must adhere to that mandate

D: Performance presentation

E: Preservation of confidentiality – unless:

- 1. The information concerns illegal activities
- 2. Disclosure is required by law
- 3. The client permits disclosure

IV: Duties to Employers

A: Loyalty

B: Additional compensation arrangements

C: Responsibilities of supervisors

V: Investment Analysis, Recommendations and Actions

A: Diligence and reasonable basis – members/candidates must:

- 1. Exercise diligence, independence and thoroughness in analyzing investments, making recommendations and taking action
- 2. Have a reasonable/adequate basis for analysis, recommendation and action

Guidance:

The application of this Standard depends on the investment philosophy adhered to, members' roles in the investment decision-making process, and the resources/support provided by employers – these action dictate the level of diligence and investigation, and the thoroughness of research required

Level of Research - Should consider:

- Firm's financial results / operating cycle / business cycle stage
- Fees / historical funds of mutual fund
- Limitations of quantitative models
- Determine if peer group comparisons for valuations are needed
- Also:
  - Encouraged to review second/third party research
  - Should be able to explain quantitative research methods
  - Members must ensure their a procedures in place to review external advisers
  - Member doesn't have to decline with be identified with a report even if they don't agree, if there is a reasonable and adequate basis

B: Communication with clients/prospective clients –

members/candidates must:

- 1. Disclose to clients the basic format/principles of the investment process
- 2. Use reasonable judgment in identifying which factors are important to their investment analyses, recommendations and actions and communicate these to clients
- 3. Distinguish b/w fact and opinion in presentation to client

Guidance:

Proper communication w/ clients is critical, members must:

- Distinguish b/w fact and opinion and explain the basics of securities in their reports
- Illustrate the investment decision-making process utilized
- Communicate risk factors and potential gains/losses
- Explain the limitations of quantitative models used

C: Record retention

Guidance:

Members must maintain research records that support the reasons for the analysts' conclusion and investment action taken (7-year holding period)

- Member's who change firms must recreate the analysis documentation support their conclusions from public info or info from the company

VI: Conflicts of Interest

A: Disclosure of conflicts

Guidance:

Disclosure to clients – clients must be able to fairly judge to conflict, its motives and potential biases for themselves. This includes disclosure of broker/dealer market-making activities

- Most common COI is actual ownership of stock in companies that the member recommends or that clients hold
- Another common COI is a member's compensation/bonus structure

Disclosure to employers – members must give their employers enough info to judge the impact of the conflict; also take reasonable steps to avoid conflict

B: Priority of transactions

Guidance:

Client transactions take precedence over personal transactions and over

transactions made on behalf of the member's firm

C: Referral fees

Guidance:

Members must inform employers, clients, and prospects of any benefit received for referrals of customers and clients, allowing them to evaluate the full cost of the service as well as any potential partiality.

VII: Responsibilities as a CFA Institute Member or CFA Candidate

A: Conduct as members/candidates in the CFA program

Do not cheat on, nor compromise the validity of, the CFA exam

B: Reference to CFA Institute, the CFA Designation, and the CFA Program

Do not make promotional promises or guarantees tied to the CFA designation. Do not:

- Over-promise individual competence
- Over-promise investment results in the future

LOS 3.a: Explain why the GIPS (Global Investment Performance Standards) were created, what parties the GIPS apply to, and who is served by the standards

GIPS are a set of ethical principles based on a standardized, industry-wide approach. Investment firms can voluntarily follow GIPS in their presentation of historical investment results to prospective clients. These standards seek to avoid misrepresentations of performance.

- GIPS apply to investment management firms are attempt to serve both prospective and existing clients

LOS 3.b: Explain the construction and purpose of composites in performance reporting

A composite is a grouping of individual discretionary portfolios representing a similar investment strategy, objective or mandate (EG: 'large cap growth stocks')

- Reporting on the performance of composites gives clients and prospects information about the firm's success in managing various types of securities or results for various investment styles
- A composite must include all portfolios (current and past) that the firm has managed in accordance with a particular strategy
    - The firm should identify a portfolio's composite before its performance is known

LOS 3.c: Explain the requirements for verification

Verification – requirements:

- A third party performs verification. The third party must attest that:
  - The firm has complied with all GIPS requirements for composite construction on a firm-wide basis
  - The firm's processes and procedures are established to present performance in accordance with the calculation methodology required by GIPS, the data requirements of GIPS, and in the format required by GIPS

Verification – recommendations

- Firms are encouraged to pursue independent verification
- "[Insert name of firm) has been verified for the periods [insert dates) by [name of verifier). A copy of the verification report is available upon request."

The GIPS

LOS 4.a: Describe the key features of the GIPS and the fundamentals of compliance

GIPS Objectives:

- To obtain global acceptance of calculation and presentation standards in a fair, comparable format w/ full disclosure
- To ensure consistent, accurate investment performance data in areas of reporting, records, marketing and presentations
- To promote fair competition among investment management firms in all markets w/o unnecessary entry barriers for new firms
- To promote global self-regulation

Key Characteristics of GIPS

- To claim compliance, an investment management firm must define its 'firm' – should reflect the 'distinct business entity' held out to clients
- GIPS are ethical standards for performance presentation which ensure fair representation of results and full disclosure
- Include all fee-paying, discretionary portfolios is composites (min. 5 yrs). After presenting 5 yrs of compliance data, the firm must add annual performance each year going forward for min 10 yrs
- Firms are required to use certain calculation/presentation standards and make specific disclosures
- Input data must be accurate
- GIPS contain both required and recommended (encouraged) provisions
- Firms are encouraged to present all pertinent additional information
- No partial compliance, only full compliance can be claimed
- Follow local laws for cases in which local standards conflict w/ GIPS, but disclose the conflict
- Supplemental PE and real estate provisions contained in GIPS are to be

applied to those asset classes

Fundamentals of compliance contain both requirements & recommendations:

- Definition of the firm – requirements:
  - Apply GIPS firm-wide
  - Firm must be defined as a distinct business unit
  - Total firms assets include total market value of discretionary and non-discretionary assets
  - Include asset performance of sub-advisors
  - If a firm changes its organization, historical composite results cannot be changed
- Definition of the firm – recommendations:
  - Include the broadest definition of the firm (including all geographical offices marketed under same brand)
- Document policies and procedures – requirements
  - Document, in writing, policies/procedures the firm uses to comply with GIPS
- Claim of compliance – requirements
  - Compliance statement: "[Insert name of firm] has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS)"
  - No partial compliance
  - No statements referring to calculation methodologies used in a composite presentation as being 'in accordance with GIPS'
- Firm fundamental responsibilities – requirements
  - Firms must provide a compliance presentation to all prospects
  - Provide a composite list/description to all prospects per request
  - Provide (per client request) a compliant presentation and composite description for any composite on the firm's list
  - When jointly marketing, be sure to separate GIPS compliance from non GIPS compliance
  - Must comply w/ requirements, encouraged to do so w/ recommendations

LOS 4.b: Describe the scope of the GIPS standards w/ respect to an investment firm's definition and historical performance record

The definition of the firm, for GIPS compliance, must be the corporation, subsidiary, or division that is held out to clients as a business entity – all geographic locations should be included

- A firm must initially present a min of 5 yrs of compliant performance presentation for the firm and each composite (unless it has existed for < 5 yrs) then compliant performance must remain for 10 yrs going forward
- Firms may present periods of noncompliant performance immediately prior to compliant performance history (as long as it isn't post Jan 2000)

LOS 4.c: Explain how the GIPS are implemented in countries w/ existing standards for performance reporting and describe the appropriate response when GIPS and local regulations conflict

Firms that previously presented performance in compliance with a particular Country Version of GIPS (CVG) may claim GIPS compliance for any CVG-compliant results prior to January I, 2006.

- Firm must follow country specific regulations ahead of GIPS, but must disclose the conflict

LOS 4.d: Describe the nine major sections of the GIPS

0. Fundamentals of Compliance:

- A: definition of the firm
- B: documentation of firm policies and procedures w/ respect to GIPS compliance
- C: complying with GIPS updates
- D: claiming compliance in the appropriate manner
- E: appropriate verification statement when a third-party verifier is employed

1. Input Data

- Input data should be consistent in order to establish full, fair and comparable investment performance presentations

2. Calculation Methodology

- Certain methodologies are required for portfolio return calculations and certain other methodologies are required for composite return calculations – uniformity in methodology is essential

3. Composite construction

- Creation of meaningful, asset-weighted composites is important to achieve a fair presentation
    - Composite performance is based on the performance of one or more portfolios w/ the same investment strategy
    - Composite returns are the asset-weighted average of the returns on the portfolios included in each composite

4. Disclosures

- The firm must disclose information about the presentation and policies adopted by the firm so that the raw numbers presented in the report are understandable to the user

5. Presentation and Reporting

- Investment performance must be presented according to GIPS requirements

6. Real Estate

- Certain provisions apply to all real estate investments regardless of the level of control the firm has over management of the investment
    - These provisions apply regardless of whether the asset is producing revenue or there is leverage involved in the investment

7. Private Equity

- Private equity investments must be valued according to the GIPS Private Equity Valuation Principles (Appendix D, unless investment in an open-end or evergreen fund)
    - PE investments include all investments in companies that are not publicly traded, regardless of their stage of business development
        - EG: VC investments, ownership of a previously public company, mezzanine financing as well as limited partnership shares in such investments and fund-of-funds investments

8. Wrap Fee / Separately Managed Account (SMA) Portfolios

- For these portfolios, some of the requirements/recommendations in sections 0 – 5 are supplemented or replaced by the requirements specified in this section

QUANTITATIVE METHODS

The Time Value of Money

When an investment is subject to compound interest, the growth in the value of

the investment from period to period reflects not only the interest earned on the original principal amount, but also on the interest earned on the previous period's interest earnings – the interest on interest.

- TVM applications frequently call for determine the Future Value of an investment's cash flow
    - Computing FV involves projecting cash flows forward on the basis of an appropriate compound interest rate
    - Computation of the Present Value works in the opposite direction
    - Computation of FV and PV is useful when comparing investment alternatives
- TVM problems can be best visualized with a time line of the cash flows

LOS 5.a: Interpret interest rates as required rates of return, discount rates, or opportunity costs

Interest rates are our measure of the TVM, although risk differences in financial securities lead to differences in their equilibrium interest rates.

- Equilibrium interest rates are the **required rate of return** for a particular investment, in the sense that the market rate of return is the return that investors and savers require to get them to willingly lend their funds
    - Interest rate = **discount rate**
    - Interest rates can also be viewed as **opportunity cost** of current consumption

LOS 5.b: Explain an interest rate as the sum of a real risk-free rate, and premiums that compensate investors for bearing distinct types of risk

The **real risk-free rate** of interest is a theoretical rate on a single-period loan that has no expectation of inflation in it. Since expected inflation in future periods is not zero, the rates we observe on US T-bills (for example) are risk-free rates, but not real rates of return. T-bill rates are **nominal risk-free** because they contain an in action premium:

Nominal risk-free rate = real risk-free rate + expected inflation rate

Securities may have one or more types of risk, and each added risk increases the required rate of return on the security. These types of risks are:

- Default risk – the risk that a borrower will not make the promised payments in a timely manner
- Liquidity risk – the risk of receiving less than fair value for an investment if it must be sold for cash quickly
- Maturity risk – the prices of longer-term bonds are more volatile than shorter-term bonds; longer-term bonds therefore have more maturity risk

Required interest rate on a security = nominal risk-free rate + default risk premium

+ liquidity premium + maturity risk premium

LOS 5.c: Calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding

Financial institutions usually quote rates as stated annual interest rates, along w/ a compounding frequency, as opposed to quoting the rates as periodic rates – the rate of interest earned over a single compounding period

- EG: bank will quote a savings rate as 8%, compounded quarterly, rather than 2% per quarter
- The rate of interest actually realized as a result of compounding is the known as the **effective annual rate (EAR)**
    - EAR represents the annual rate of return actually being earned after adjustments have been made for different compounding periods

$EAR = (1 + periodic\ rate)^m - 1$

- Periodic rate = state annual return / m
- M = number of compounding periods per year

The EAR increases as the compounding frequency increases

- The limit of a shorter and shorter compounding periods is called continuous compounding
    - $EAR\ (w/\ continuous\ compounding) = e^r - 1$

LOS 5.d: Solve time value of money problems for different frequencies of compounding

See examples; remember $FV = PV(1+r)^t$

LOS 5.e: Calculate and interpret the FV and PV of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV) and a series of unequal cash flows.

Future Value of a Single Sum

FV of single cash flow:

$FV = PV(1 + I/Y)^N$

- PV = present value
- I/Y = rate of return per compounding period
- N = total number of compounding periods
- In this expression, the investment involves a single cash outflow, PV, occurring at t = 0

Present Value of a Single Sum

$$PV = \frac{FV}{(1+\frac{I}{Y})}$$

Annuities

An **annuity** is a stream of equal cash flows that occur at equal intervals over a given period.

- Ordinary annuity – characterized by cash flows that occur at the end of each compounding period
- Annuity due – payments/receipts occur at the beginning of each period

The difference b/w single sum and annuity TVM problems is that instead of solving for the PV or FV of a single cash flow, we solve for the PV or FV of a stream of equal periodic cash flows, where the size of the periodic cash flow is defined by the payment (PMT) variable on your calculator

Future Value of an Annuity Due

Set calculator to BGN (beginning-of-period) mode

- $FVA_D = FVA_O \times (1 + I/Y)$

Present Value of an Annuity Due

With an annuity due, there is one less discounting period since the first cash flow occurs at t = 0, and thus is already its PV. This implies, that all else equal, the PV of an annuity due will be greater than the PV of an ordinary annuity

There are two ways to compute the PV of an annuity due:

- Set calculator to BGN mode and input relevant variables (N, I/Y, PMT), or,
- Treat cash flow stream as an ordinary annuity over N compounding periods, and multiply the resulting PV by (1 + periodic compounding rate I/Y)
    - $PVA_D = PVA_O \times (1 + I/Y)$

Present Value of a Perpetuity

A perpetuity is a financial instrument that pays a fixed amount of money at set intervals over an infinite period of time (ie: a perpetual annuity).

- The discount factor of a perpetuity is 1/r (r = the appropriate rate of return)
- $PV_{perpetuity} = PMT / (I/Y)$

PV and FV of Uneven Cash Flow Series

These cash flows aren't annuities because the cash flows are different every year

- Essential can be viewed as a stream of annual single sum cash flows

    - Thus to find PV/FV of the stream we just sum the PVs/FVs of the individual cash flows
        - Can also use NPV/NFV function on calculator for PV/FV

Solving Time Value of Money Problems When Compounding Periods Are Other Than Annual

More frequent compounding periods will have an impact on FV and PV calculations

- An increase in the frequency of compounding increases the effective rate of interest, and also increases the FV of a given cash flow and decreases the PV of a given cash flow
- Can leave calculator in annual compounding mode and instead adjust the rate (I/Y) and the number of periods (N)

LOS 5.f: Demonstrate the use of a time line in modeling and solving time value of money problems

Loan Payments and Amortization

Loan amortization is the process of paying off a loan w/ a series of periodic loan payments, whereby a portion of the outstanding loan amount is paid off, or amortized, with each payment

- A long term loan is usually paid off over time w/ a series of equal, periodic loan payments, and each payment includes the repayment of principal and an interest charge
- Amortization table: Period; beginning balance; payment; interest component (1); principal component (2); ending balance (3)
    - Interest component (1) = beginning balance x periodic i/r
    - Principal component (2) = payment – interest (1)
    - Ending balance (3) = period's beginning balance – principal component (2)
- *See examples (pp. 119-123)

Funding a Future Obligation

There are many TVM applications where it is necessary to determine the size of the deposit(s) that must be made over a specific period in order to meet a future liability. EG:

- (1): setting up a funding program for future college tuition
- (2): the funding of a retirement program

The objective is generally to determine the necessary payment size

The Connection Between Present Values, Future Values, and Series of Cash Flows

As explained earlier, for annuities w/ uneven cash flows, the sum of the present

values of the cash flows is the present value of the series. The sum of the future values of a series of cash flows is the future value of the series.

The **cash flow additivity** principle refers to the fact that the present value of any stream of cash flows equals the sum of the present value of the cash flows.

- If we have two series of cash flows, the sum of the present values of the two series is the same as the present values of the two series taken together, adding cash flows that will be paid at the same point in time

Discounted Cash Flow Applications

LOS 6.a: Calculate and interpret the net present value (NPV) and the internal rates of return (IRR) of an investment

The **NPV** of an investment is the present value of expected cash inflows associated with the project less the present value of the project's expected cash outflows, discounted at the appropriate cost of capital. The procedure is:

- Identify all costs (outflows) and benefits (inflows) of an investment
- Determine the appropriate discount rate / opportunity cost for the investment
- Using the discount rate, find the PV of each cash flow
    - Inflows are positive and increase NPV
    - Outflows are negative and decrease NPV
- Compute the NPV (the sum of the DCFs)

$$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

- CF_t = expected net cash flow at time t
- N = estimate life of the investment
- r = discount rate
- NPV is the PV of the cash flows less the initial outlay (at t=0)

The **IRR** is defined as the rate of return that equates the PV of an investment's expected benefits (inflows) w/ the PV of its costs (outflows). Equivalently, the IRR may be defined as the discount rate for which the NPV of an investment is zero

- Calculating IRR only requires that we identify the relevant cash flows of an investment

$$0 = CF_0 + CF_1/(1+IRR) + CF_2/(1+IRR)^2 + \ldots + CF_N/(1+IRR)^N$$

- In the majority of IRR applications to capital budgeting, the initial cash flow, CF_0, represents the initial cost of the investment opportunity, and is therefore a negative value.
    - A discount rate less than the IRR will result in a positive NPV

    - A discount rate greater than the IRR will result in a negative NPV
    - When discount rate = IRR, NPV = 0

LOS 6.b: Contrast the NPV rule to the IRR rule, and identify problems associated with the IRR rule

The NPV decisions rules:

- Accept project w/ positive NPV, as shareholder wealth will increase
- Reject project w/ negative NPV, as shareholder wealth will decrease
- When two projects are mutually exclusive, accept the one w/ the higher positive NPV

The IRR decision rules:

- Accept projects w/ an IRR that is greater than the firm's (investor's) required rate of return
- Reject projects w/ an IRR that is less than the firm's (investor's) required rate of return

Problems associated with the IRR method:

- When the acceptance/rejection of a project has no effect on the acceptance/rejection of another, the two projects are considered independent
    - When only one of two projects may be accepted, they are mutually exclusive
- For mutually exclusive projects, the NPV and IRR methods can give conflicting project rankings
    - This can happen when the project's initial costs are of different sizes or when the timing of the cash flows are different
- **Always choose the result that maximizes shareholder wealth the most**
    - **Always select the project w/ the greatest NPV when NPV and IRR provide conflicting results**
- The NPV method assumes reinvestment of a project's cash flows at the opportunity cost of capital, while the IRR method assumes that the reinvestment rate is the IRR
    - The discount rate used with the NPV approach represents the market-based opportunity cost of capital and is the required rate of return for the shareholders of the firm

LOS 6.c: Calculate and interpret holding period return (total return)

A holding period can be any period of time. The **holding period return (HPR)** is simply the percentage change in the value of an investment over the period it is held

- If the asset has cash flows, we refer to the return as the **total return**

HPR = [(ending value + cash flow received) / beginning value] – 1

LOS 6.d: Calculate and compare the money-weighted and time-weighted rates of return of a portfolio and evaluate the performance of portfolios based on these measures

The **money-weighted return** applies the concept of IRR to investment portfolios. It is defined as:

- The internal rate of return on a portfolio, taking into account all cash inflows and outflows
    - The beginning value of the account is an inflow, as are all deposits into the account
    - All withdrawals from the account are outflows, as is the ending value

The **time-weighted return** measures compound growth. It is defined as:

- The rate at which $1 compounds over a specified performance horizon
- Time-weighting is the process of averaging a set of values over time
- The annual time-weighted return for an investment may be computed by performing the following steps:
    1. Value the portfolio immediately preceding significant additions or withdrawals. Form subperiods over the evaluation period that correspond to the dates of deposits / withdrawals
    2. Compute the HPR of the portfolio for each subperiod
    3. Compute the product of (1+HPR) for each period to obtain a total return for the entire measurement period
        a. If the total investment period is > 1 year, you must take the geometric mean of the measurement period return to find the annual time-weighted rate of return
- **The time-weighted rate of return is the preferred method of performance measurement because it isn't affected by the timing of cash inflows and outflows**

If funds are contributed to an investment portfolio just before a period of relatively poor portfolio performance, the money-weighted rate of return will tend to be lower than the time-weighted rate of return (vice versa).

- The use of time-weighted return removes these distortions and thus provides a better measure of a manager's ability to select investments over the period

LOS 6.e: Calculate and interpret the bank discount yield, holding period yield, effective annual yield and money market yield for US Treasury bills and other money market instruments

Pure discount T-bills are quoted differently from US government bonds, T-bills are quoted on a bank discount basis, which is based on the face value of the instrument instead of the purchase price. The **bank discount yield (BDY)** is

computed using the following formula:

$r_{BD} = D/F \times 360/t$

- r_{BD} = annualized yield on a bank discount basis
- D = dollar discount (difference b/w face value and purchase price)
- F = face value
- t = number of days until maturity
- 360 = bank convention of days in a year

The key distinction of bank discount yield is that it expressed the dollar discount from the face value as a fraction of the face value, not the market price of the instrument.

The yield quoted on a bank discount basis is not representative of the return earned by an investor for the following reasons:

- Bank discount yield annualizes using simple interest and ignores the effect of compound interest
- Bank discount yield is based on the face value of the bond, not its purchase price – investment returns should be evaluated relative to the amount invested
- Bank discount yield is annualized based on a 360-day year, not 365

Holding period yield (HPY), or holding period return, is the total return an investor earns b/w the purchase date and the sale / maturity date. HPY is calculated using:

$HPY = (P_1 + D_1)/P_0 - 1$

- P_0 = price of investment
- P_1 = price received for investment at maturity (face value)
- D_1 = interest payment (distribution)

*T-bills are pure discount instruments; therefore make no interest payments ($D_1 = 0$)

The **effective annual yield (EAY)** is an annualized value based on a 365-day year that accounts for compound interest. It is calculated using:

$EAY = (1 + HPY)^{365/t} - 1$

The **money market yield (or CD equivalent yield)** is equal to the annualized holding period return, assuming a 360-day year.

- Using the money market yield makes the quoted yield on a T-bill comparable to yield quotes for interest bearing money market instruments that pay interest on a 360-day basis
- Formula is:

$r_{MM} = HPY \times (360/t)$

Given the BDY, the money market yield may be calculated as:

$r_{MM} = (360 \times r_{BD}) / [360 - (t \times r_{BD})]$

LOS 6.f: Convert among holding period yields, money market yields, effective annual yields, and bond equivalent yields

Once we have established HPY, EAY or r_{MM}, we can use one as a basis for calculating the other two. Remember:

- The **HPY** is the actual return an investor will receive if the money market instrument is held until maturity
- The **EAY** is the annualized HPY on the basis of a 365-day year and incorporates the effects of compounding
- The $\mathbf{r_{MM}}$ is the annualized yield that is based on price and a 360-day year, and does not account for compounding

The **bond equivalent yield** refers to 2 x the semiannual discount rate. This convention stems from the fact that yields on US bonds are quoted as twice the semiannual rate, because the coupon interest is paid in two semiannual payments

- First convert yield into an effective semiannual yield
- The multiply by 2

Statistical Concepts and Market Returns

LOS 7.a: Distinguish between descriptive statistics and inferential statistics, between a population and a sample, and among the types of measurement scales.

Descriptive statistics are used to summarize the important characteristics of large data sets. **Inferential statistics** pertain to procedures used to make forecasts, estimates or judgments about a large set of data on the basis of the statistical characteristics of a smaller set

A **population** is defined as a set of all possible members of a stated group. A **sample** is a subset of the population of interest.

Types of measurement scales (NOIR)

- *Nominal scales* – level of measurement w/ the least information; observations are classified or counted w/ no particular order
- *Ordinal scales* – represent a level of measurement where every observation is assigned to one of several categories, then these categories are ordered w/ respect to a specified characteristic
- *Interval scales* **–** provide relative ranking, like ordinal scales, plus the

assurance that the differences b/w scale values are equal (zero does not indicate absence of what is being measured)
- *Ratio scales* – most refined level of measurement; provide ranking and equal differences b/w scale values and also have a true zero point as the origin

LOS 7.b: Define a parameter, a sample statistic and a frequency distribution

A measure used to describe a characteristic of a population is referred to as a **parameter**. A **sample statistic** is used to measure a characteristic of a sample.

A **frequency distribution** is a tabular presentation of statistical data that aids the analysis of large data sets. They assigned statistical data to specified groups or intervals, and such data can be measured w/ any type of measurement scale

How to construct a frequency distribution:

1. *Define the intervals* – an interval (class) is the set of values that an observation may take on. The range of values must have an upper and lower limit and be all-inclusive and non-overlapping (intervals must be mutually exclusive). Must also use an appropriate number of intervals
2. *Tally the observations* – assign data to appropriate intervals
3. *Count the observations* – the number of observations assigned to each interval must be tallied; the frequency is the actual number of observations that fall within a given interval

* The interval w/ the greatest frequency is the modal interval

LOS 7.c: Calculate and interpret relative frequencies and cumulative relative frequencies, given a frequency distribution

Relative frequency is calculated by dividing the absolute frequency of each return interval by the total number of observations (result is a %age). It is possible to compute the **cumulative absolute frequency** and **cumulative relative frequency** by summing the absolute or relative frequencies at the lowest interval and progressing to the highest.

LOS 7.d: Describe the properties of a data set presented as a histogram or a frequency polygon

A **histogram** is a graphical presentation of the absolute frequency distribution

- Simply a bar chart of continuous data
- Intervals on the horizontal axis, absolute frequencies on the vertical

To construct a **frequency polygon**, the midpoint of each interval is plotted on the horizontal axis, and the absolute frequency for that interval is plotted on the vertical axis. Each point is then connected w/ a straight line.

LOS 7.e: Calculate and interpret measures of central tendency, including

the population mean, sample mean, arithmetic mean, weighted average or mean, geometric mean, harmonic mean, median and mode

Population mean: all observed values in the population (N) are summed and divided by the number of observations in the population

Sample mean: all observed values in the sample (n) are summed and divided by the number of observations in the sample

Both the population and sample means are examples of **arithmetic means** – the sum of the observation values divided by the number of observations

- All interval and ratio data sets have an arithmetic mean
- A data set has only one arithmetic mean
- The sum of the deviations of each observation in the data set from the arithmetic mean is always zero
- Unusually large/small values can have a disproportionate effect on the arithmetic mean

The computation of a **weighted mean** recognizes that different observations may have a disproportionate influence on the mean

$$\bar{X}_W = \sum_{i=1}^{n} w_i X_i = (w_1X_1 + w_2X_2 + \ldots + w_nX_n)$$

where:
$X_1, X_2, \ldots X_n$ = observed values
$w_1, w_2, \ldots w_n$ = corresponding weights associated with each of the observations such that $\Sigma w_i = 1$

The return of a portfolio is the weighted average of the returns of the individual assets within the portfolio

- Asset weights are market weights, the market value of each asset relative to the market value of the portfolio

The **median** is the midpoint of a data set when the data is arranged in ascending / descending order. Outliers do not affect the median.

The **mode** is the value that occurs most frequently in the data set. A data set can have more than one mode or even no mode. When a distribution has one value that appears most frequently, it is called **unimodal** (then bimodal, trimodal, etc).

The **geometric mean** is often used when calculating investment returns over multiple periods or when measuring compound growth rates.

$$G = \sqrt[n]{X_1 \times X_2 \times \ldots \times X_n} = (X_1 \times X_2 \times \ldots \times X_n)^{1/n}$$

This equation has a solution only if the product under the radical sign is non-negative. When calculating the geometric mean for a returns data set, must remember to add one to each value under the square root, then subtract one

$$1 \quad R_G = \sqrt[n]{(1 \quad R_1) \times (1 \quad R_2) \times \ldots \times (1 \quad R_n)}$$

where:
R_t = the return for period t

from the result (following diagram missing + signs)

* The geometric mean is always < or = to the arithmetic mean

A **harmonic mean** is used for certain computations, such as the average cost of shares purchased over time. The harmonic mean is calculated as = $\frac{N}{\sum_{i=1}^{N}\frac{1}{X_i}}$

For values that aren't all equal, harmonic mean < geometric mean < arithmetic mean. This mathematical fact is the basis for the claimed benefit of purchasing the same dollar amount of mutual fund shares each month or each week ('dollar cost averaging').

LOS 7.f: Calculate and interpret quartiles, quintiles, deciles and percentiles

A **quantile** is the general term for a value at or below which a stated proportion of the data in a distribution lies.

- *Quartiles* – the distribution is divided into quarters
- *Quintile* – the distribution is divided into fifths
- *Decile* – the distribution is divided into tenths
- *Percentile* – the distribution is divided into hundredths

Any quantile can be expressed as a percentile.

The formula for the position of the observation at a given percentile, y, with n data points sorted in ascending order is:

$L_y = (n + 1) * y/100$

* Quantiles and measures of central tendency are known as **measures of location**

LOS 7.g: Calculate and interpret 1) a range and a mean absolute deviation and 2) the variance and standard deviation of a population and sample

Dispersion is defined as the variation around the central tendency. A common theme in finance/investments is reward v variability, where central tendency is a measure of reward and dispersion is a measure of risk.

The **range** is a relatively simple measure of variability, but when used w/ other measures it provides extremely useful information.

Range = maximum value – minimum value

The **mean absolute deviation (MAD)** is the average of the absolute values of the deviations of individual values from the arithmetic mean

$$MAD = \frac{\sum_{i=1}^{n}|X_i - \bar{X}|}{n}$$

* Must use absolute values because the sum of actual deviations from the arithmetic mean is zero

The **population variance** is defined as the average of the squared deviations from the mean. The population variance (σ^2) uses the values for all members of a population and is calculated using the following formula:

$$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$$

Generally this result is converted into a **population standard deviation**, which is simply the square root of the population variance.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X - \mu)^2}{N}}$$

The **sample variance**, s^2, is the measure of dispersion that applies when we are evaluating a sample of *n* observations from a population

$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$$

* Note that the formula for sample variance uses the denominator (n-1), one less than the sample size, where as population variance uses the denominator N, representing the entire population. The use of the entire sample size n will underestimate the population parameter σ^2. Also sample mean is used instead of population mean.

The **sample standard deviation** is simply the square root of the sample variance.

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

LOS 7.h: Calculate and interpret the proportion of observations falling within a specified number of standard deviations of the mean using Chebyshev's inequality

Chebyshev's inequality states that for any set of observations, whether sample or population data and regardless of the shape of the distribution, the percentage of the observations that lie within *k* standard deviations is *at least* $1 - 1/k^2$ *for all* $k > 1$

LOS 7.i: Calculate and interpret the coefficient of variation and the Sharpe ratio

A direct comparison b/w two or more measures of dispersion can be difficult due to a large difference in means, etc. This is overcome using a relative measure of dispersion. **Relative dispersion** is the amount of variability in a distribution relative to a reference point or benchmark. Relative dispersion is commonly measured with the **coefficient of variation (CV)**, which is computed as:

$$CV = \frac{s_x}{\bar{X}} = \frac{\text{standard deviation of x}}{\text{average value of x}}$$

CV measures the amount of dispersion in a distribution relative to the distribution's mean.

- This allows for direct comparisons of dispersion across different sets of data
- CV is used to measure the risk (variability) per unit of expected return (mean)

The **Sharpe ratio** is widely used for investment performance measurement and measures *excess* return per unit of risk.

$$\text{Sharpe ratio} = \frac{\bar{r}_p - r_f}{\sigma_p}$$

- r_p = portfolio return
- r_f = risk-free return
- σ_p = standard deviation of portfolio returns

Portfolios w/ large positive Sharpe ratios are preferred to portfolios w/ smaller ratios because it is assumed that rational investor prefer high return and dislike risk

Sharpe ratio limitations

- If two portfolios have negative Sharpe ratios, it is not necessarily true that the higher Sharpe ratio implies superior risk-adjusted performance
    - Increasing risk moves a negative Sharpe ratio closer to zero
- The Sharpe ratio is useful when standard deviation is an appropriate measure of risk, however many portfolios have asymmetric return distributions

LOS 7.j: Explain skewness and the meaning of a positively or negatively skewed return distribution

A distribution is symmetrical if it is shaped identically on both sides of its mean. The extent to which a distribution is symmetrical is important because the degree of symmetry tells analysts if deviations from the mean are more likely to be positive or negative.

Skewness, or skew, refers to the extent to which a distribution is not symmetrical

- A *positively skewed* distribution is said to be skewed right because of its long upper (right) tail
- A *negatively skewed* distribution is said to be skewed left because of its long lower (right) tail

LOS 7.k: Describe the relative locations of the mean, median and mode for a unimodal, nonsymmetrical distribution

Skewness affects the mean, median and mode of a distribution

- Symmetrical distribution – mean = median = mode
- Positively skewed distribution – mode < median < mean
    - Mean is affected by outliers
- Negatively skewed distribution – mean < median < mode

LOS 7.l: Explain measures of sample skewness and kurtosis

Kurtosis is a measure of the degree to which a distribution is more or less 'peaked' than a normal distribution.

- *Leptokurtic* describes a distribution that is more peaked than a normal distribution
    - More returns clustered around the mean and more returns with large deviations from the mean (fatter tails)
    - Greater percentage of small deviations from the mean and greater percentage of large deviations from the mean
- *Platykurtic* describes a distribution that is less peaked (flatter) than a normal distribution
- *Mesokurtic* means a distribution has the same kurtosis as a normal distribution

A distribution is said to exhibit *excess kurtosis* if it has either more of less kurtosis than the normal distribution. The computed kurtosis for all normal distributions is three (3), however scientists often report excess kurtosis, which is defined as kurtosis minus 3. Therefore normal distributions have excess kurtosis of zero (0).

- Leptokurtic dist has excess kurtosis > 0, platykurtic dist has excess kurtosis < 0

In practice, most securities have returns that are not normally distributed, and exhibit both skewness and kurtosis, both of which are critical concepts for risk management.

- Generally, greater positive kurtosis and more negative skew in return distribution indicates increased risk