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$$p = \frac{\hbar}{i} \frac{\partial}{\partial x}$$

$$x = i\hbar \frac{\partial}{\partial p}$$

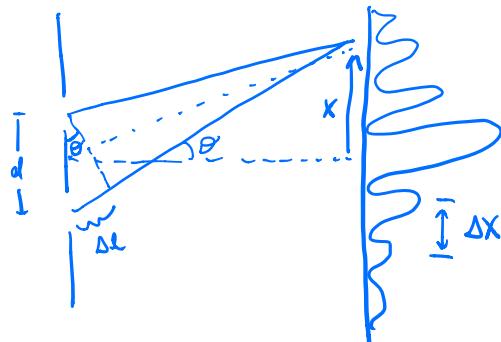
1. Wave - Particle Duality

Double slit experiment

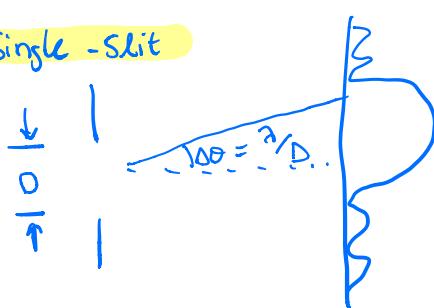
$$\Delta l = d \sin \theta = d \theta = d \tan \theta = \frac{dx}{L}$$

$$\Delta x = \frac{\lambda}{d} L \rightarrow \text{spacing of fringes}$$

$$\Delta l = \begin{cases} n\lambda & , \text{ constructive} \\ (2n+1) \frac{\lambda}{2} & , \text{ destructive} \end{cases}$$



Single - Slit



Photoelectric effect

$$E_e = h\nu - W \rightarrow \text{energy ejected e-}$$

- energy photon: $E = h\nu$
 $\nu = ck \rightarrow (\text{speed of light})(\text{wavelen}) \quad \text{wavelen} \# k = \frac{m}{\lambda}$

$$\Rightarrow \vec{p} = \hbar \vec{k}$$

Darinson - Fermer experiment

- showed e^- diffraction

de Broglie wavelength

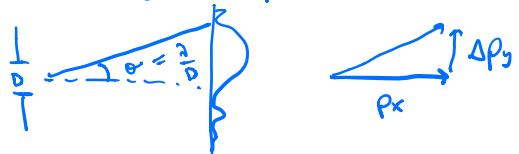
$$\lambda = \frac{h}{p}$$

$$\omega = \frac{E}{\hbar}$$

$$E_{kin} = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

Wave - Part. reconciliation

+ Uncertainty Principle (qualitative)

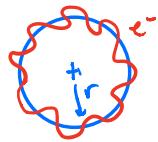


$$\rightarrow \frac{\Delta p_y}{p_x} = \frac{\lambda}{D} \quad \text{and} \quad \Delta y = D$$

$$\Rightarrow \Delta y \Delta p_y = p_x \lambda = \hbar$$

↳ qualitative intuition ONLY!
 → only applies for scale λ .

Atom Stability



$$2\pi r = n\lambda \quad (\text{Bohr-Sommerfeld quantization})$$

Since $\lambda \propto \frac{1}{p}$, now r related to p . in QM

$$E_{\text{tot}} = KE + PE = \frac{p^2}{2mr^2} - \frac{e^2}{r}$$

↪ large E const for having e^- in small radius

Wavefunction

$$\psi(\vec{r}, t) = e^{i\vec{k} \cdot \vec{r} - i\omega t}$$

↪ probability amplitude of particle's presence at \vec{r} at time t

2. The Schrodinger Equation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V(\vec{r}, t)\psi = i\hbar \frac{\partial}{\partial t} \psi \rightarrow \nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}, \vec{r} = \hat{x}\hat{y} + \hat{z}$$

↪ 1st order D.E. in time, given $V(\vec{r}, t)$, $\psi(\vec{r}, t)$ is defined
⇒ complex numbers are essential

Probability Interpretation

$$P(\vec{r}, t) = |\psi(\vec{r}, t)|^2$$

$$\int_{\text{all space}} d\vec{r} P(\vec{r}, t) = \int_{\text{all space}} d\vec{r} |\psi(\vec{r}, t)|^2 = 1$$

Conservation Laws

→ particles are not created, destroyed
↪ the solution ψ must be
CONTINUOUS + DIFFERENTIABLE
↪ $\nabla \psi$ is also cont. for non
singular potential.

Probability Flux

$$\vec{J} = \frac{i\hbar}{2m} [\psi^* \vec{\nabla} \psi - (\vec{\nabla} \psi^*) \psi]$$

Superposition Principle

- If ψ_1 and ψ_2 are solutions to S.E. → Then $\psi_1 + \psi_2$ is as well

Free particle

→ not subject to any potentials S.E. → $-\frac{\hbar^2}{2m} \nabla^2 \psi = i\hbar \frac{\partial}{\partial t} \psi$

plane wave solution

$$e^{i\vec{k} \cdot \vec{r} - i\omega t}$$

$$\hbar \omega(k) = \frac{\hbar^2 k^2}{2m}$$

→ to satisfy
Energy
momentum
relation

$$\hookrightarrow \psi(\vec{r}, t) = A_1 e^{i\vec{k}_1 \cdot \vec{r} - i\omega(k_1)t} + A_2 e^{i\vec{k}_2 \cdot \vec{r} - i\omega(k_2)t} + A_3 \dots$$

Fourier Decomposition

→ adding together large number of solutions. → $\Sigma \rightarrow \int$

$$\psi(\vec{r}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k g(k) e^{i(k \cdot r - \omega t)}$$

→ $g(k)$ probability amplitude of having some momentum

• this is the general solution of free particle S.E.

$$0 \quad \psi(x, 0) = C e^{ikx} \cdot e^{-k^2/2} \quad \text{for 1D, } t=0$$

$|g(k)|^2 \rightarrow$ probability
particle has momentum
 $p = \hbar k \rightarrow k = p/\hbar$

Heisenberg Uncertainty Principle
→ fully mathematical result
 $\Delta x \Delta p \geq \hbar$

$$g(k) = \frac{1}{(2\pi)^{2/3}} \int d^3r e^{-i k \cdot r} \psi(\vec{r}, 0)$$

Fourier transform
given we know
 $\psi(\vec{r}, 0)$

3. Stationary States

Lecture 3, 4

Generally, $V(\vec{r}, t) = V(\vec{r}) \rightarrow$ time independent potential

Then $|\psi(\vec{r}, t)|^2$ is time-independent as well. $\left(\frac{d^2}{dt^2} = 0 \right)$

Take $\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})$, then $\hat{H} \phi(\vec{r}) = E \phi(\vec{r})$

↳ For $E = \hbar \omega \rightarrow$ eigenvalue and energy of state and,

$$\psi(\vec{r}, t) = \phi(\vec{r}) e^{-i Et/\hbar}$$

General Sol to time dependent S.E.

$$\psi(x, t) = \sum_{n=1}^{\infty} C_n \phi_n(x) e^{-i E_n t / \hbar}$$

→ superposition of stationary states

Principle of Spectral Decomposition

- any measurement of system yields one of the eigenstates
- any initial state is represented as combination of eigenfunctions.

$$\psi(x, 0) = \sum_n C_n \phi_n(x)$$